User's Guide to S-Parameters Explorer 2.0

EE Circle Solutions

December 2010

Contents

Part I

New Features in S-Parameter Explorer Version 2.0

First of all, a high-quality scientific plotting package, MjoGraph, is used for presenting the S-parameter data. We are grateful for the permission granted by Dr. Makoto Tanahashi, the author of MjoGraph. Interested users are referred to his web-site for more information: `http://www.ochiailab.dnj.ynu.ac.jp/mjograph/`. A separate guide for the plotter is also provided along the EE Circle tool. However, due to some compatibility issues, the Mjograph plotting module is only functional for the Mac OS-X users, and might be available at a later time for other OS platforms.

Version 1.0 of the S-Parameter Explorer had very limited capabilities of exploring. In Version 2.0, many transformations and manipulations of the S-parametrs are possible. New features include:

- S-parameter port re-indexing
- Reducing an S-parameter block by either leave un-used ports OPEN or terminated at the default reference impedance
- Renormalization with an arbitrary (positive) value of new reference impedance
- Allow separation of each port into two circuit nodes, with options to combine selected group of the local reference nodes
- Generating S-parameters from single or multiply-coupled transmission-line models
- Extract RLGC parameters from a (2N)-port S-parameter block representing N coupled transmission-lines
- A built-in circuit simulator to let the user to (a) compose a sub-circuit using the existing S-blocks and lumped L/R/C elements, and (b) generate a new S-parameter block

Acknowledgement

- S-Parameter Explorer is written 100% in Java^{TM} `http://www.oracle.com/technetwork/java/index.html`.
- The following Java numerical packages are instrumental building blocks for S-Parameter Explorer
  - JFreechart: a comprehensive plotting package. `http://www.jfree.org`
  - PtPlot: A plotting package as a part of the Ptolemy at UC Berkeley. `http://ptolemy.berkeley.edu/java/ptplot/`
  - Colt: a free Java toolkit at CERN for high performance computing. `http://dsd.lbl.gov/òhoschek/colt/`
  - JAMA (JAva MAtrix package): containss common matrix routines. `http://math.nist.gov/javanumerics/jama/`
  - Jampack (JAva Matrix PACKage): matrix solver for complex matrices. `ftp://math.nist.gov/pub/Jampack/Jampack/AboutJampack.html`
- Additional credits to Java libraries can be found in the user's guide for MjoGraph
- This document is prepared with LaTeX (`http://www.latex-project.org/`) and edited with LyX (`http://www.lyx.org/`)

Part II

GUI Overview

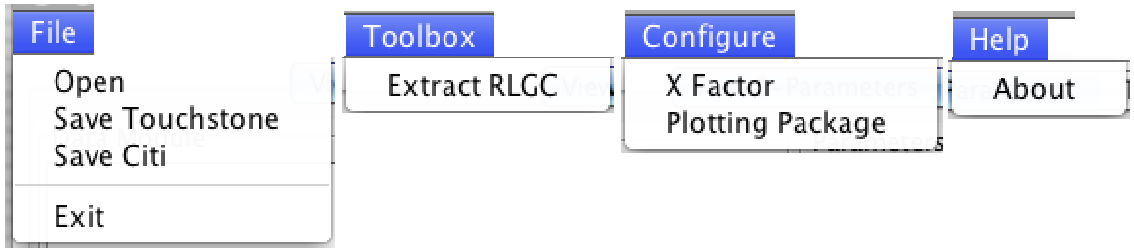

Figure 0.2.1: Menu elements in S-Parameter Explorer 2.0

This part of the document explains the use of the tool through the GUI elements.

0.1 Tips about This Tool's Memory Consumption

- Occassionaly, the tool might run into limitation of heap memory used by this Java application
  - When some S-blocks have many number of ports (>10) and large number of frequency points
- The choice of the internal data resampler, linear or spline, may make a significant difference in memory consumption
  - The default linear sampler works well if the resampling frequency points are close to the readily defined frequency grid
  - The spline sampler might be necessary when the original frequency grid is sparse compared to the targeted frequency sequence

In case the user runs into the memory limitation problem, consider launching the tool inside a terminal (cd into the tool's installation directory) with one of the supplied batch scripts. In a batch mode, the user can manually adjust the heap size allocated to Java with the command-line options like "-Xmx512m" setting the maximum heap size to 512MB.

0.2 Menu Elements

- File
  - Open: a dialog will pop up and let user to choose S-parameter files in either Touchstone or Citi format
  - Save Touchstone: a dialog will pop up and let user to save the currently selected S-parameter data in a Touchstone file
  - Save Citi: a dialog will pop up and let user to save the currently selected S-parameter data in a Citi file
  - Exit: quit the entire program
- Toolbox
  - The program will attemp to extract RLGC parameters from the selected (2N)-port S-parameter data. Upon success, results will be saved in an HSPICE RLGC file.
- Config
  - X Factor: set the scaling factor such that the next-read S-parameter files will have its frequency sequence normalized by the factor
  - Plotting Package: allow user to choose the plotting packages from (a) MjoGraph, (b) JFreeChart, and (c) PtPlot

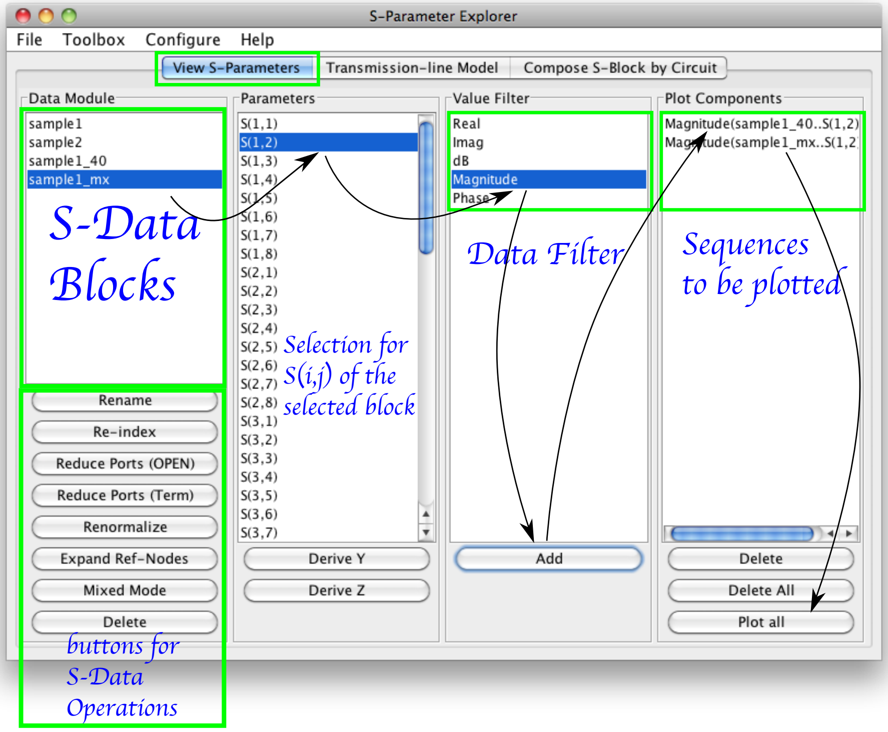

Figure 0.3.1: Overview of the "View S-Parameter" taab.

- Resampler Choice: either Linear or Spline can be chosen

- Help
  - About: gather information about the computer and the license status

0.3 The "View S-Parameters" Tab

More detailed information about various manipulations of S-parameter blocks can be found in Chapter 1. The upper left list area contains all the S-parameter blocks currently stored. The buttons below the "S-Data Blocks" listing area are designed to trigger certain operations (see Figure 0.3.2):

- Rename: allow an S-parameter data block to be renamed
- Re-index: A pop-up dialog will collect a new sequence of indices (need to be a permutation of the N indices where N is the number of ports). The new data block will essentially be the same only with the indexing reshuffled
- Reduce Ports (OPEN): A pop-up dialog will ask the user to specify the indices for ports being kept. All the unused ports are left OPEN
- Reduce Ports (Term): A pop-up dialog will ask the user to specify the indices for ports being kept. All the unused ports are terminated with the reference impedance
- Renormalize: A pop-up dialog will ask the user to provide a positive number to be the new reference impedance
- Expand Ref-Nodes: A pop-up dialog will be presented to configure the way the local reference nodes are exposed
- Mixed Mode: A pop-up dialog will collect N indices, and the program will assign differential pairs by picking index-pairs in order

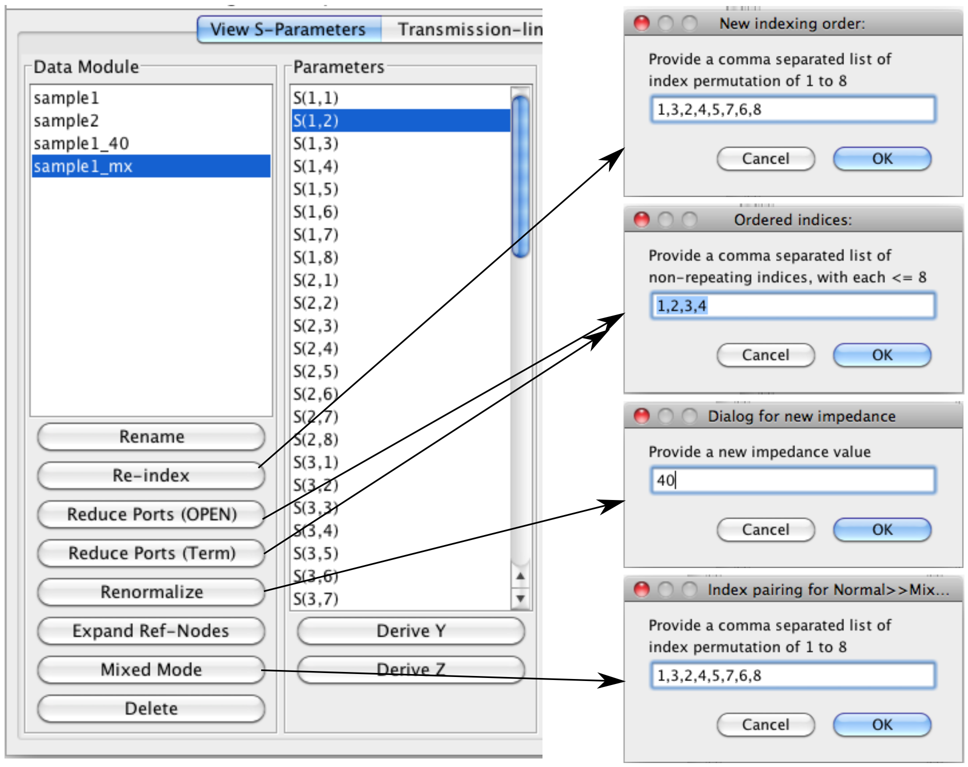

Figure 0.3.2: Dialogs triggered by the S-parameter manipulation buttons.

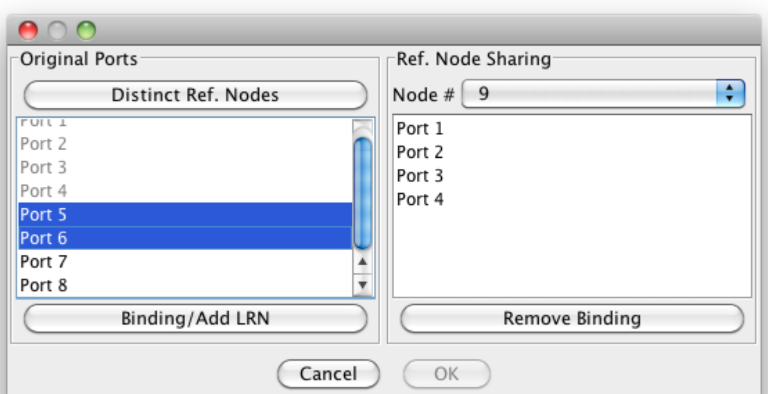

Figure 0.3.3: Dialog for assigning local reference nodes.

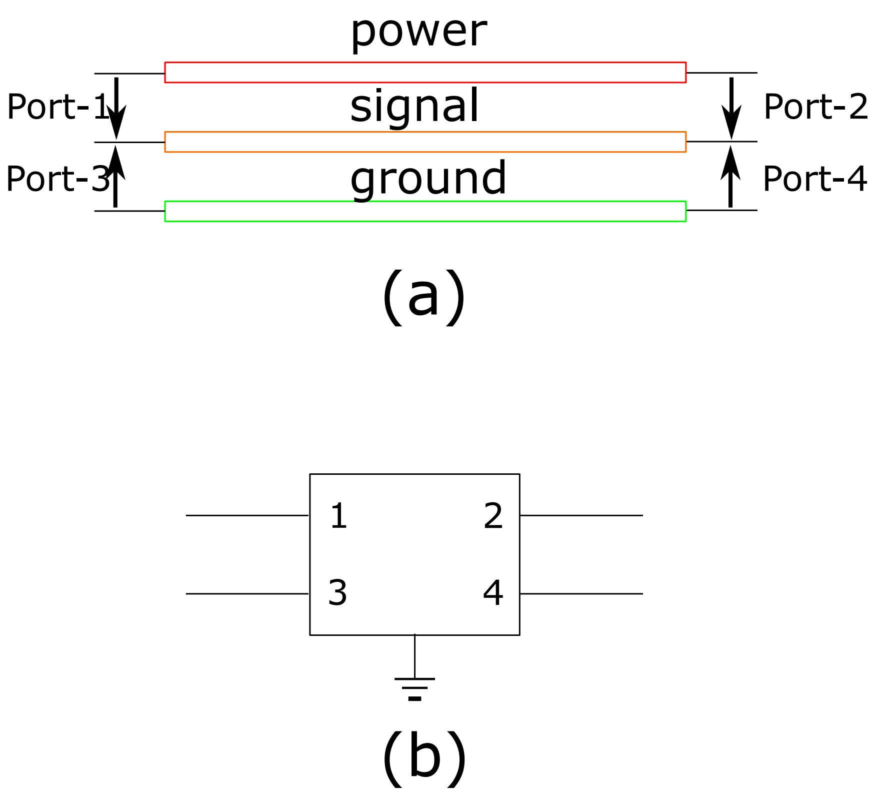

Figure 0.3.4: A 4-port model from a EM simulation.

- Delete: cause the selected data block to be removed

The example dialog for configuring local reference nodes (LRNs) is shown in Figure 0.3.3. The underlying concepts are explained below:

- Each port voltage can be expanded into the difference between two nodes, each is referencing to the global GND node where the potential in ZERO

$$v_p^{(i)} = v_+^{(i)} - v_-^{(i)},\ i = 1, 2, ..., N$$

- When the "Distinct Ref Nodes" button is clicked, every port is split into two separate nodes. This results in a (2N)-port network, where the (N+i)-th node is the the local reference node of the original Port-i, for $i = 1, 2, ..., N$

- Alternatively, the user can merge some of the N LRNs and assigned to a new node. Upon clicking the "Binding/Add LRN", a new node will be created for the group of LRNs. Such LRN grouping actions need to continue until all the local reference nodes are exhausted (and all Port indices are dimmed in the left listing area)

- The combobox (dropdown list) in the upper right contains all indices for the currently created node for LRNs. When the displayed LRN nodal index is changed, the listing area below will be updated to show the corresponding LRNs of the original ports.

- The "Remove Binding" button will will the currently shown LRN-node to be disbanded

- The "OK" button will be lit up only when the LRN assignments are complete. A click of it will cause the program to generate a $(N + N_1)$-port that is a alternative representation of the original N-port data. Here $N_1 \leq N$

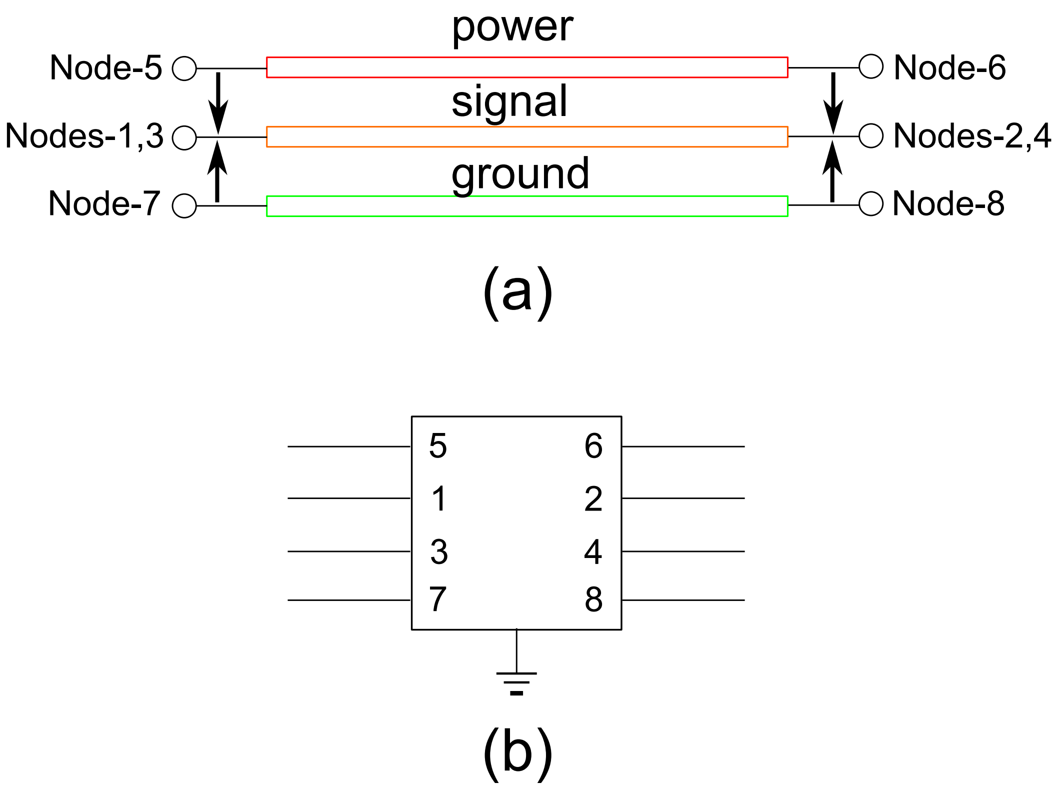

Figure 0.3.5: LRN manipulation applicable to the stripline model.

0.3.1 Example of LRN

As an example, observe a stripline in a printed circuit environment, where the signal line has two possible current return (voltage reference) structures - one is ground (at bottom) and the other is power (on top). An accurate modeling approach of such structure is to assign two ports at each end of the signal line, referencing to the power and ground planes, respectively as shown in Figure 0.3.4(a). The simulation results are captured into a 4-port S-block. Common SPICE-like circuit simulators work with nodes and the concept of ports associated with S-parameters may not be properly mapped, a typical representation for the stripline model would be like Figure 0.3.4(b). The fact that Ports1,3 and Ports 2,4 need to have different reference nodes can not be reflected.

The actual S-parameter data can be found in the $SPEX20/sampleData directory and named “stripline_2ref.s4p”. The S-parameter data can be properly transformed to be compatible with the SPICE-like simulators, yet reflect the physics properly:

- Use menu Open to read the “stripline_2ref.s4p” data
- Select the “stripline_2ref” in the left-most list in the “View S-Parameters” tab
- Click the “Expand Ref-Node” button, a dialog will popup
- Click the “Distinct Ref. Nodes” button within the dialog. Click “OK”
- Now, there is a new S-block named “stripline_2ref_lrn”

The new S-block is best represented by the schematic symbol in Figure 0.3.5(b) where all the reference node are now exposed and accessible. It can be easily blended with other circuit elements to investigate system's subtle and complex behaviors such as - the influence of the decpoupling capacitors (these caps can be tied between Nodes 5,7 and Nodes 6,8). An example schematic is shown in Figure 0.3.6.

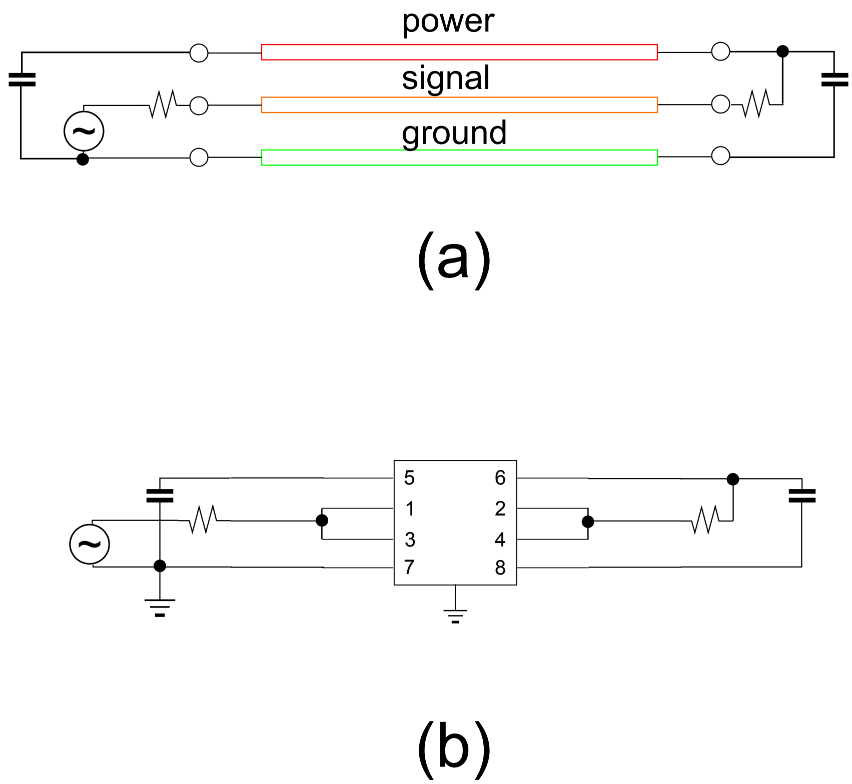

Figure 0.3.6: Model used to investigate signal on a stripline referencing to power and GND planes.

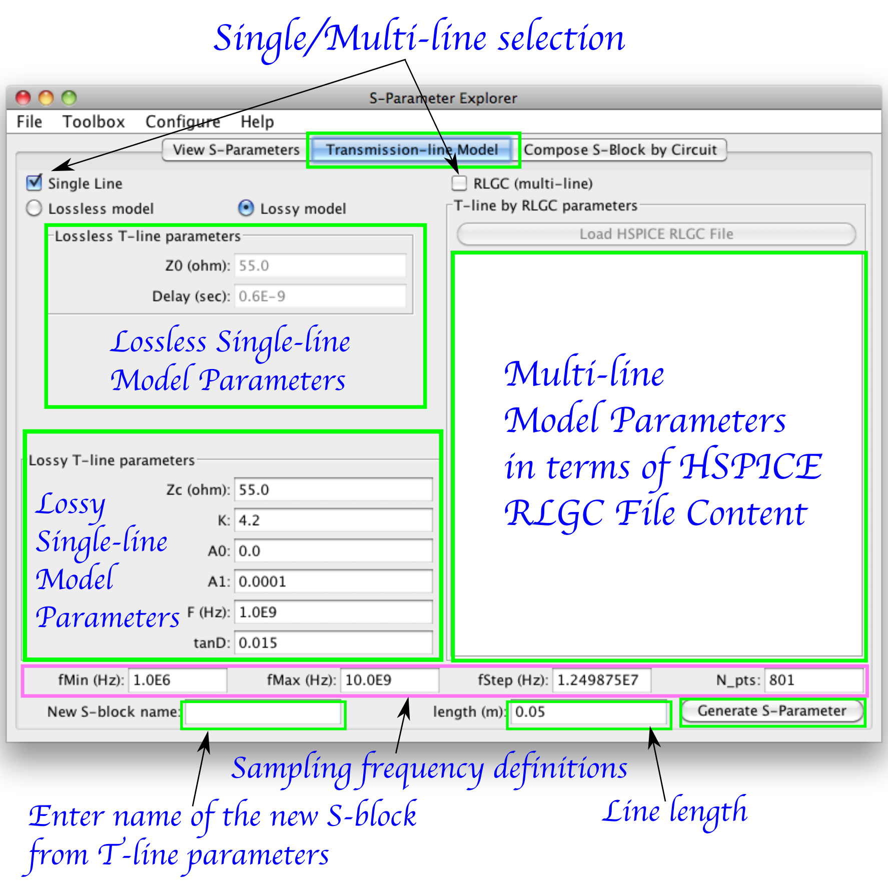

Figure 0.4.1: Overview of the “Transmission-line Model” tab.

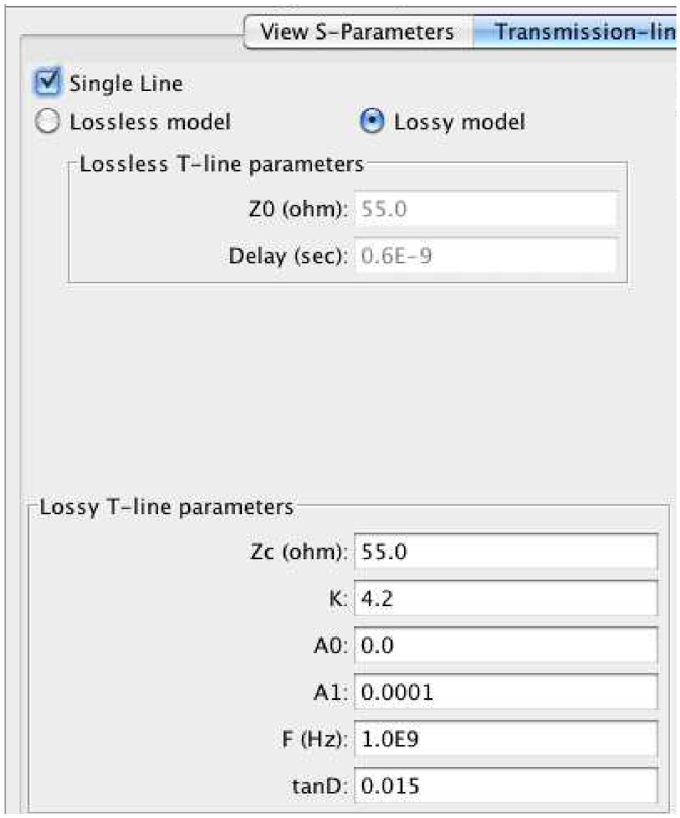

Figure 0.4.2: Input parameter fields for single lossless and lossy transmission-line.

0.4 The “Transmission-line Model” Tab

Besides loading S-parameters that are generated external to this program through simulations or measurements, it is now possible to generate S-parameters within the tool’s own environment. One way of generating S-parameters is to derive from transmission-line models. For more background information and technical details, please read Chapter 2. By activating the “Transmission-line Model” tab, the user can:

- Choose single-line model
  - Lossless single-line model, which needs only two parameters: characteristic impedance and total delay
  - Lossy single-line model in ADS TLINP style. Please read 2.2.3 for the parameter definitions. In particular, the A0, A1 terms are associated wit the DC and skin losses of the conductor, as shown in Eqn.(2.2.3)
- Choose multi-line model
  - The user can load an HSPICE RLGC file for a W-element. Details for the single-line version of the W-element model can be found in Sub-section 2.2.2. More information can be found in [7]. Such files can also be calculated from the EE Circle Trace Analyzer

Examples of the single- and multi-conductor transmission-line input area are shown in Figures 0.4.2 and 0.4.3, respectively.

0.5 The “Compose S-Block by Circuit” Tab

Users need to understand the following concept and assumptions before this tool can be effectively used:

- Component Library contains library components which may be

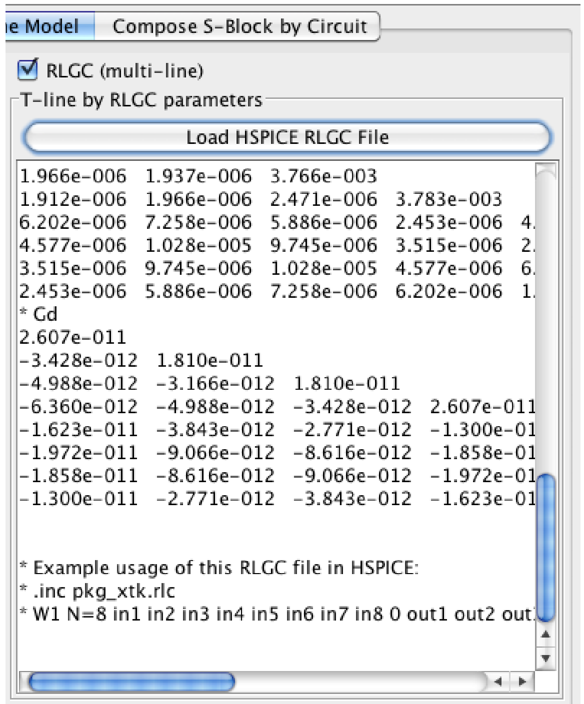

Figure 0.4.3: Input parameter fields for multi-conductor transmission-line.

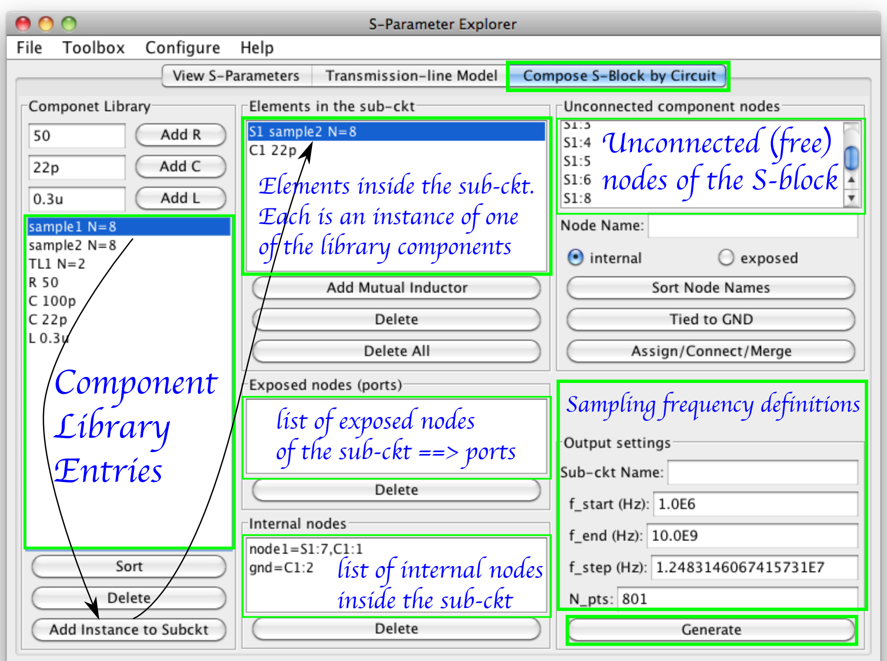

Figure 0.5.1: Overview of the “Compose S-Block by Circuit” tab.

  - An S-parameter block
  - A resistor with a distictive value
  - A capacitor with a distinctive value
  - An inductor with a distictive value

- User can only compose one sub-ckt at a time
- Elements in a the sub-ckt must be instances (or copies) of the components in the library
- Each element contains a number of indexed nodes, and the nodes are identified by <elementName>:<nodeIndex>
- Some nodes in the sub-ckt are made accessible to the outside world, therefore called exposed nodes, and the rest of the nodes are called internal nodes

A typical composition of a sub-circuit goes through the following steps:

- Develop an adequate component library, especially for S-blocks that might ever be used in any sub-ckt
- Sketch out a schematic diagram, on a piece of scratch paper, with element named, nodal indices assigned
- Start creating the sub-ckt, by clicking the "Add instance to Subckt" button a number of times, such that elements are added
- Mutual inductors are not generated from any library components. They can be generated when two inductors are selected.
- User works with the list of unconnected node in the upper right corner
  - Select the nodes (from various elements) that are supposed to be connected.
    * Multiple selections of the list elements can be achieved by holding down the Ctrl or Command key while clicking
    * Choose either "internal" or "exposed" radio button
    * Optionally type in a customized node-name
    * Click "Assign./Connect/Merge" button to make a connection if the node is not grounded.
    * Click the "Tie to GND" button to attach the selected nodes to GND
    * The nodal connection process is repeated until the list of "unconnected node" is exhausted
- Inside the "Output setting" panel, user can provide
  - Name of the sub-ckt, which will used used to label the new S-block upon successful circuit calculation
  - Frequency minimum and maximum values
  - Frequency step-size and number of poits are interdependent. If user wants to take either as the preferrred choice, just press ENTER after entering the desired value
- Press the "Generate" button. A popup message should be displayed if the computation of the S-parameters for the newly composed sub-ckt comes through

0.5.1 Example: Composing a Differential Channel Model

Figure 0.5.2 shows a simple differential channel composed of interconnect elements like vias, traces and load capacitors.

- The S-parameters for vias, S1 and S3, are supplied by the "sampleData/via2.s4p"
- The S-parameters for coupled microstrips are provided by the "ustrip2_5in.s4p" file
- The task of building such a circuit will be realized in the "Compose S-Block by Circuit" tab

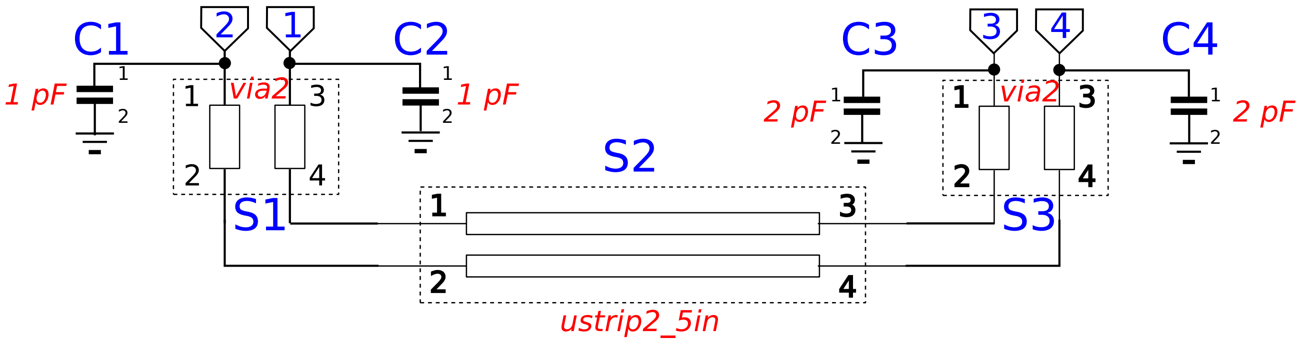

Figure 0.5.2: A differential channel model consisting of vias, traces and capacitors.

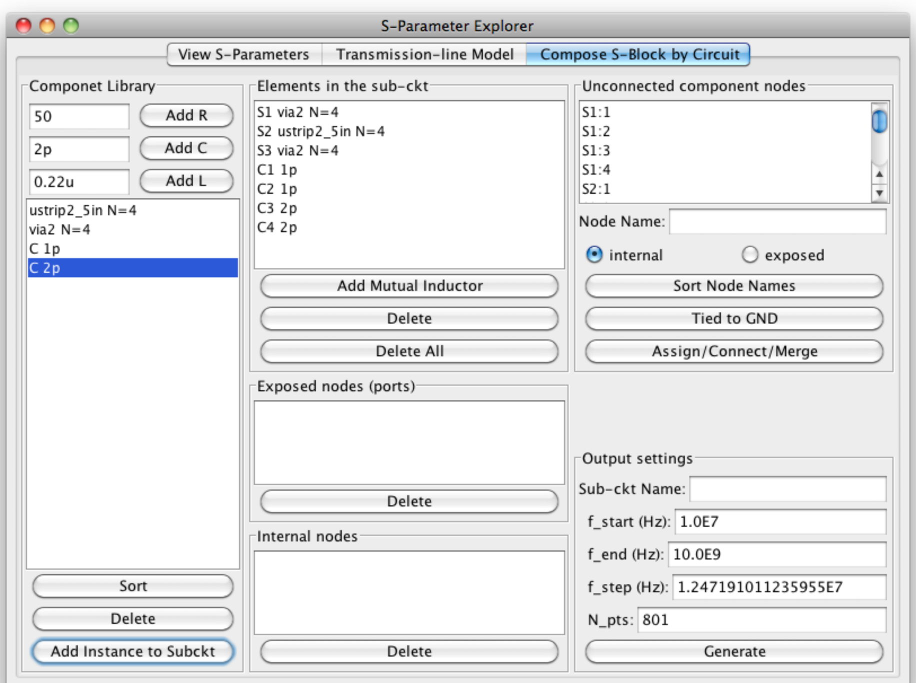

Figure 0.5.3: Preparing the component library, and creating elements for the sub-ckt.

The actual steps of composing the differetial channel model are:

- Load the two files “via2.s4p” and “ustrip_5in” from the $SPEX/sampleData directory
- Click the “Compose S-Block by Circuit” tab title. There should be two library components listed “ustrip2_5in N=4” and “via2 N=4”
- Edit the value next to the “Add C” button, by entering “1p”, then click “Add C”. This cause an “C 1p” component to be added into the library
- Edit the value next to the “Add C” button, by entering “2p”, then click “Add C”. This cause an “C 2p” component to be added into the library
- Choose lib component “via2”, click “Add Instance to Subckt” once. This causes an “S1 via2 N=4” item to be added to the list of elements
- Choose lib component “ustrip2_5in”, click “Add Instance to Subckt” once. This causes an “S2 ustrip2_5in N=4” item to be added to the list of elements
- Choose lib component “via2”, click “Add Instance to Subckt” once. This causes an “S3 via2 N=4” item to be added to the list of elements
- Choose lib component “C 1p”, click “Add Instance to Subckt” button twice. This cause two capacitor instances C1 and C2 to be added to the elements
- Choose lib component “C 2p”, click “Add Instance to Subckt” button twice. This cause two capacitor instances C3 and C4 to be added to the elements
- Make connection for the exposed nodes. Set the radio button “exposed node” active
  - Select “S1:3” and “C2:1” in the list of “Unconnected element nodes”, click “Assign/Connect/Merge”
  - Similar connectors made for “S1:1,C1:1”, “S3:1,C3:1” and “S3:3,C4:1”
- Make connections for the internal nodes. Set the radio button “internal node” active
  - Make connections “S1:2,S2:2”, “S1:4,S2:1”, “S2:3,S3:2” and “S2:4,S3:4”
  - Select all nodes left in the unconnected node list C1:2, C2:2, C3:2, and C4:2. Click “Tied to GND”

The tool should now look like Figure 0.5.4. User only needs to fill out the Sub-ckt Name, say “diffChannel” and set the frequency parameters. A click of the “Generate” should trigger the calculation for the overall channel model. If successful, a popup message should inform the user that a new S-block is available.

By employing the mixed-mode transformations to the via, trace and overall channel models, we can plot the insertion losses (or gains) in Figure 0.5.5. Of course, there are many more possibilities that the S-parameter comosition method can be used. Since the newly created S-block is automatically added to the component library, user can compose complex model hierarchically using the simple interface.

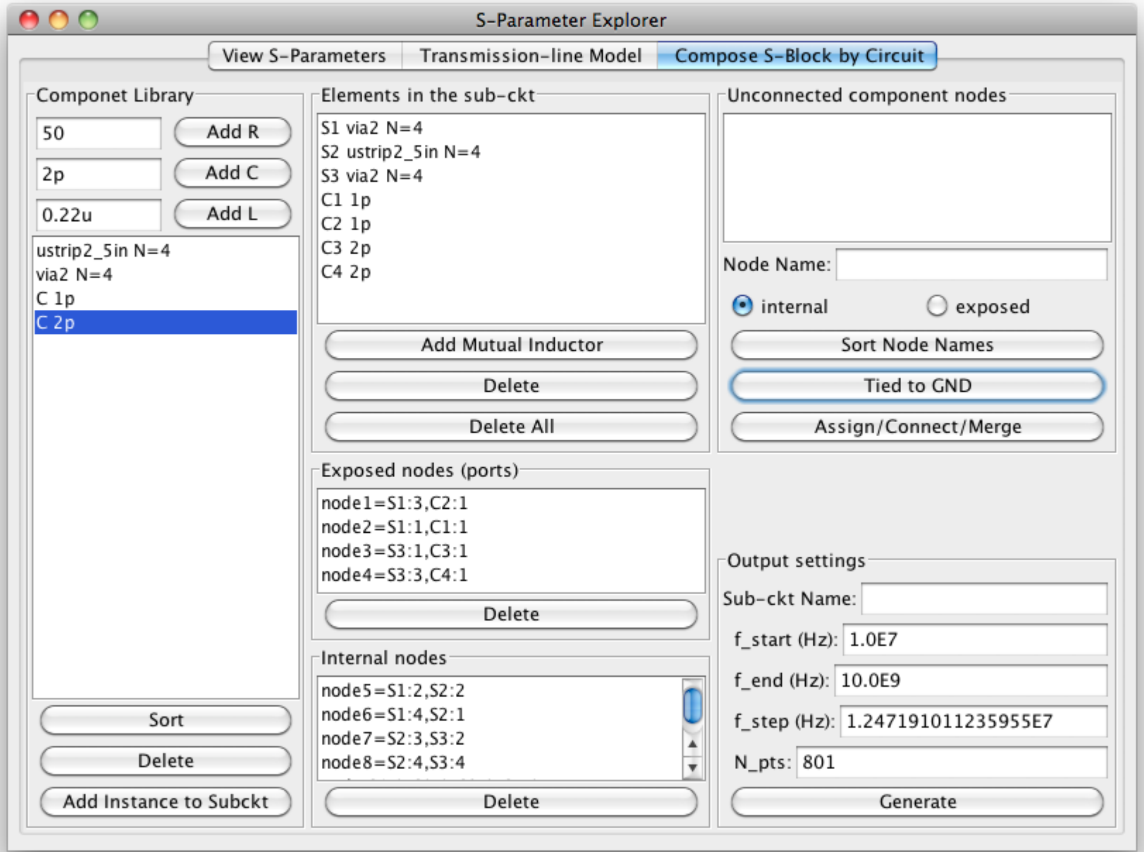

Figure 0.5.4: View of the tool when composition of the sub-ckt is completed.

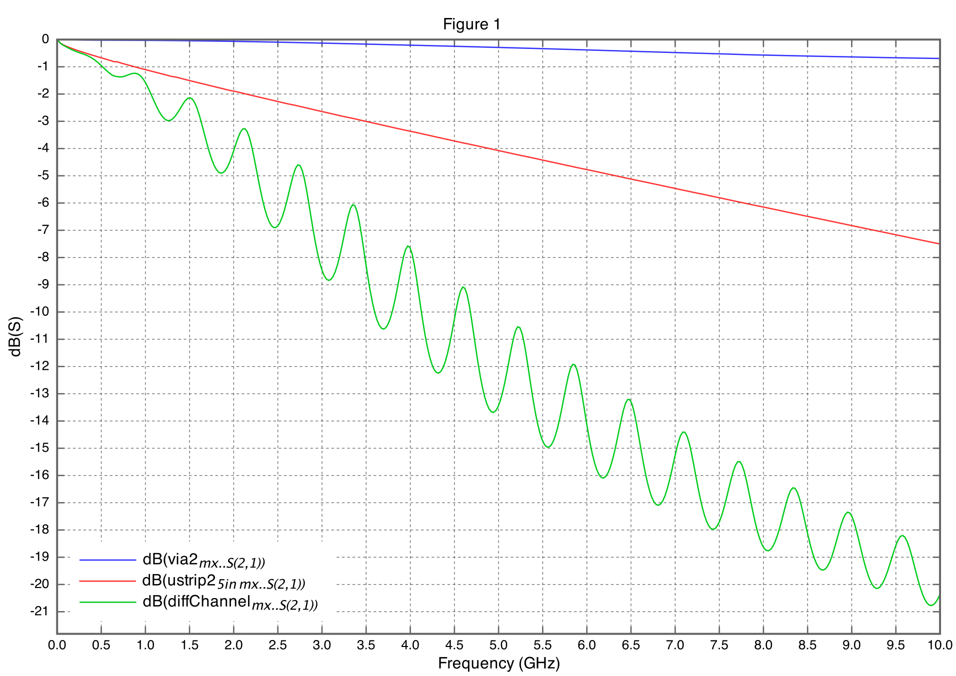

Figure 0.5.5: Plots showing the insertion losses of vias, traces and the entire channel.

Part III

Theories and Technical Notes

This part of the document is for the inquisitive users who like to know what's under the hood of the S-Parameter Explorer 2.0.

Chapter 1

S-Parameter Block Manipulations

1.1 The AB=BA Theorem

Based on the reciprocity in electromagnetics, network transfer functions (impedance, admittance, scattering-parameters) associated with isotropic media are symmetric. To better facillitate some derivations in the following sections, the "AB=BA theorem" is stated below:

Fact 1. *Matrice compositions of* $\alpha\mathbf{A}+\beta\mathbf{B}$, $\mathbf{AB}$, $\mathbf{A}^{-1}$ *are symmetric, provided* $\mathbf{A}$ *and* $\mathbf{B}$ *are both symmetric,* $\mathbf{A}$ *is non-singular, and* α, β *are scalar constants.*

Theorem 2. *If the matrix-form of a network function* $\mathbf{C}$ *is expressed as the product of two other matrices,* $\mathbf{C} = \mathbf{AB}$*. If* $\mathbf{A}$, $\mathbf{B}$*, and* $\mathbf{C}$ *are all known to be symmetric, then the alternative form* $\mathbf{C} = \mathbf{BA}$ *is also valid. In other words,* $\mathbf{C} = \mathbf{AB} = \mathbf{BA}$.

Proof. By the properties of the symmetric matrices, it is easily shown that $\mathbf{AB} = \mathbf{C} = \mathbf{C}^T = (\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T = \mathbf{BA}$. □

Combining Fact 1 with Theorem 2, we can easily write results like

$$\mathbf{C} = (\alpha\mathbf{A}+\mathbf{I})(\mathbf{B}+\beta\mathbf{I})^{-1} = (\mathbf{B}+\beta\mathbf{I})^{-1}(\alpha\mathbf{A}+\mathbf{I})$$

when $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are all known to be symmetric. Where $\mathbf{I}$ is the identity matrix, and α, β are scalar constants.

1.2 Basic Relations for an S-Parameter Block

Sacttering parameters [1], or S-parameters in short, specifies the frequency-domain relation between the in-going (a_i) and out-going (b_i) wave variables with

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \ldots & \ldots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \tag{1.2.1}$$

where a_i and b_i variable have the unit of square-root of power, and are related to the familiar current and voltage variables by

$$a_i = \frac{V_{p,i} + I_{p,i}Z_0}{\sqrt{Z_0}},\ b_i = \frac{V_{p,i} - I_{p,i}Z_0}{\sqrt{Z_0}} \tag{1.2.2}$$

The constant Z_0 is the reference port impedance. It should be noted that:

- Z_0 is a positive constant.
- The value of Z_0 is typically 50, especially when the S-parameters come from measurement with a vector network analyzer (VNA).

- In the more general formulations of the S-parameters, ports can have different reference impedances. In such cases, one constant value Z_0 will be replaced by $Z_{0,i}$'s in the definition equations of (1.2.2).
- Without the use of Z_0, a network can be adequately characterized by either the impedance **Z** or the admittance **Y** matrix. However, the **Z** matrix suffers singularities $Z_{ii} = \infty$ for OPEN ports; and the **Y** matrix runs into trouble with $Y_{ii} = \infty$ for SHORT ports.
- The use of the "redundant" Z_0 avoids the singularities in **Z** and/or **Y** network transfer function matrices.
- There are cases that Z_0 can be a value different than the ubiquitous constant 50. In particular, S-parameters generated by electromagnetic field solvers can be assigned an arbitrary (positive) value.
- In the original definition of the S-parameters, it is implied that each port of the network starts with a segment of transmission-line [2]. Each port has an implicit characteristic impedance value $Z_{c,i}$. In fact, assuming transmission-line at ports ensures that the voltages/currents can be defined (under the TEM approximation of the electromagnetic fields) in frequency-domain. However, it is a common source of confusion to get the concept of port reference impedance mixed up with idea of the port characteristic impedance when the notations are not carefully chosen.

1.3 S-Parameter Renormalization

An authoritative reference article on the subject of S-parameter renormalization is [3]. In that article, different results were given depending on either the pseudo-wave or power-wave formulation was used. Readers should be aware that only the peudo-wave formulation is applicable to the modern day usage (at least within the scope of this document) of the S-parameters.

1.3.1 Pseudo-wave Formulation (The Correct Formulation)

The incident and reflected waves at Port-i are defined by

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\Re\{Z_i\}|}}, \ b_i = \frac{V_i - Z_i I_i}{2\sqrt{|\Re Re\{Z_i\}|}}.$$

The conversion of the initial matrix **S** to the new matrix **S′** is specified by

$$\mathbf{S'} = \mathbf{A}^{-1}(\mathbf{S} - \mathbf{R})(\mathbf{I} - \mathbf{RS})^{-1}\mathbf{A},$$

where

$$\mathbf{R} = \begin{bmatrix} \frac{Z_1' - Z_1}{Z_1' + Z_1} & 0 & \cdots & 0 \\ 0 & \frac{Z_2' - Z_2}{Z_2' + Z_2} & \cdots & 0 \\ & & \ddots & \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{Z_N' - Z_N}{Z_N' + Z_N} \end{bmatrix},$$

and

$$\mathbf{A} = \begin{bmatrix} \sqrt{\frac{\Re\{Z_1\}}{\Re\{Z_1'\}}} \left|\frac{Z_1'}{Z_1}\right| \frac{1}{Z_1' + Z_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\frac{\Re\{Z_2\}}{\Re\{Z_2'\}}} \left|\frac{Z_2'}{Z_2}\right| \frac{1}{Z_2' + Z_2} & \cdots & 0 \\ & & \ddots & \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \sqrt{\frac{\Re\{Z_N\}}{\Re\{Z_N'\}}} \left|\frac{Z_N'}{Z_N}\right| \frac{1}{Z_N' + Z_N} \end{bmatrix}.$$

1.3.2 Positive Real-Valued Reference Impedances

When all ports share the same reference impedances, and the "starting" and "end" reference impedances are both positive real valued, then

$$\mathbf{R} = \frac{Z_0' - Z_0}{Z_0' + Z_0}\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{Z_0'-Z_0}{Z_0'+Z_0} \end{bmatrix} = \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{I},$$

and

$$\mathbf{A} = \frac{1}{Z_0' + Z_0}\sqrt{\frac{Z_0'}{Z_0}}\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} = \frac{1}{Z_0' + Z_0}\sqrt{\frac{Z_0'}{Z_0}}\mathbf{I}.$$

Then

$$\mathbf{S}' = (\mathbf{S} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{I})(\mathbf{I} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{S})^{-1}. \tag{1.3.1}$$

1.3.3 Alternative Derivation

The admittance matrix can be derived from S-parameters [2] by

$$\mathbf{Y} = Z_0^{-1}(\mathbf{I} + \mathbf{S})^{-1}(\mathbf{I} - \mathbf{S}), \tag{1.3.2}$$

and conversely,

$$\mathbf{S}' = (\mathbf{I} - Z_0\mathbf{Y})(\mathbf{I} + Z_0\mathbf{Y})^{-1}.$$

Here $\mathbf{I}$ is the identity matrix. By Theorem 2 for symmetric matrices,

$$\mathbf{Y} = Z_0^{-1}(\mathbf{I} - \mathbf{S}).(\mathbf{I} + \mathbf{S})^{-1}.$$

Under the new reference impedance

$$\mathbf{S}' = (\mathbf{I} - Z_0'\mathbf{Y})(\mathbf{I} + Z_0'\mathbf{Y})^{-1}.$$

Invoking Theorem 2 again yields

$$\mathbf{S}' = (\mathbf{I} + Z_0'\mathbf{Y})^{-1}(\mathbf{I} - Z_0'\mathbf{Y}). \tag{1.3.3}$$

Substituting Eqn.(1.3.2) into (1.3.3)

$$\begin{aligned}
\mathbf{S}' &= [\mathbf{I} + Z_0'Z_o^{-1}(\mathbf{I} + \mathbf{S})^{-\mathbf{1}}(\mathbf{I} - \mathbf{S})]^{-1}[\mathbf{I} - Z_0'Z_o^{-1}(\mathbf{I} + \mathbf{S})^{-\mathbf{1}}(\mathbf{I} - \mathbf{S})] \\
&= \{(\mathbf{I} + \mathbf{S})^{-1}[(\mathbf{I} + \mathbf{S}) + Z_0'Z_o^{-1}(\mathbf{I} - \mathbf{S})]\}^{-1}[\mathbf{I} - Z_0'Z_o^{-1}(\mathbf{I} + \mathbf{S})^{-\mathbf{1}}(\mathbf{I} - \mathbf{S})] \\
&= [(\mathbf{I} + \mathbf{S}) + Z_0'Z_o^{-1}(\mathbf{I} - \mathbf{S})]^{-1}(\mathbf{I} + \mathbf{S})[\mathbf{I} - Z_0'Z_o^{-1}(\mathbf{I} + \mathbf{S})^{-\mathbf{1}}(\mathbf{I} - \mathbf{S})] \\
&= [(\mathbf{I} + \mathbf{S}) + Z_0'Z_o^{-1}(\mathbf{I} - \mathbf{S})]^{-1}[(\mathbf{I} + \mathbf{S}) - Z_0'Z_o^{-1}(\mathbf{I} - \mathbf{S})] \\
&= [(\mathbf{I}(1 + Z_0'Z_0^{-1}) + (1 - Z_0'Z_0^{-1})\mathbf{S}]^{-1}[(\mathbf{I}(1 - Z_0'Z_0^{-1}) + (1 + Z_0'Z_0^{-1})\mathbf{S}] \\
&= [(\mathbf{I} + (1 + Z_0'Z_0^{-1})^{-1}(1 - Z_0'Z_0^{-1})\mathbf{S}]^{-1}[(\mathbf{I}(1 + Z_0'Z_0^{-1})^{-1}(1 - Z_0'Z_0^{-1}) + \mathbf{S}] \\
&= (\mathbf{I} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{S})^{-1}(\mathbf{S} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{I}).
\end{aligned}$$

One more time using Theorem 2, we have

$$\mathbf{S}' = (\mathbf{S} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{I})(\mathbf{I} - \frac{Z_0' - Z_0}{Z_0' + Z_0}\mathbf{S})^{-1}, \tag{1.3.4}$$

which is identical to Eqn.(1.3.1). The last equation has been implemented inside S-Parameter Explorer 2.0.

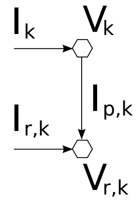

Figure 1.4.1: Relationship between the nodal current/voltage and the port-based variables.

1.4 S-Parameter for Ports and S-Parameter for Nodes

It has been discussed in the last section that the frequency-domain S-Parameters originally associated with ports, which implicitly assume transmission-line structures. A multi-conductore transmission-line system always implies a reference conductor, therefore, we can natually split the port voltage into the difference between potentials of the "signal-node" and the reference-node as shown in Figure 1.4.1. Of course the reference node needs to be preferably mapped onto the reference conductor. On the other hand, in SPICE-like circuit simulation envornment, there has to be a common reference node (GND) where the nodal potential is always ZERO. Viewing from the mathematical stand-point, the S-parameters can also be used in the context of nodal voltages.

Substituting Eqn.(1.2.2) into (1.2.1) yields

$$\begin{bmatrix} V_{p,1} \\ V_{p,2} \\ \vdots \\ V_{p,N} \end{bmatrix} - Z_0 \begin{bmatrix} I_{p,1} \\ I_{p,2} \\ \vdots \\ I_{p,N} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \dots & \dots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \left(\begin{bmatrix} V_{p,1} \\ V_{p,2} \\ \vdots \\ V_{p,N} \end{bmatrix} + Z_0 \begin{bmatrix} I_{p,1} \\ I_{p,2} \\ \vdots \\ I_{p,N} \end{bmatrix} \right),$$

or in concise form

$$(\mathbf{U} - \mathbf{S})\mathbf{V}_P = Z_0(\mathbf{U} + \mathbf{S})\mathbf{I}_P \tag{1.4.1}$$

where $\mathbf{U}$ is the identity matrix of order N. The portal admittance matrix is

$$\mathbf{Y}_P = Z_0^{-1}(\mathbf{U} + \mathbf{S})^{-1}(\mathbf{U} - \mathbf{S}).$$

Conversely, the S-parameters can be derived from nodal admittance by

$$\mathbf{S} = (\mathbf{U} - Z_0\mathbf{Y}_P)(\mathbf{U} + Z_0\mathbf{Y}_P)^{-1}.$$

The original voltage variables are defined as portal voltages, and the eventual nodal form calls for

$$\mathbf{V}_P = \mathbf{V} - \mathbf{V}_R$$

where

$$\begin{aligned} \mathbf{V}_P &= \begin{bmatrix} V_{p,1} & V_{p,2} & \dots & V_{p,N} \end{bmatrix}^T \\ \mathbf{V} &= \begin{bmatrix} V_1 & V_2 & \dots & V_N \end{bmatrix}^T \\ \mathbf{V}_R &= \begin{bmatrix} V_{1,r} & V_{2,r} & \dots & V_{N,r} \end{bmatrix}^T \\ \mathbf{I}_P &= \begin{bmatrix} I_{p,1} & I_{p,2} & \dots & I_{p,N} \end{bmatrix}^T \end{aligned}$$

then the nodal equation is

$$[\mathbf{Y}_P][\mathbf{V} - \mathbf{V}_R] = [\mathbf{I}_P]$$

or

$$[\mathbf{Z}_P][\mathbf{I}_P] = [\mathbf{V} - \mathbf{V}_R] = \begin{bmatrix} \mathbf{U} & -\mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_R \end{bmatrix}$$

where

$$\mathbf{Z}_P = \mathbf{Y}_P^{-1}.$$

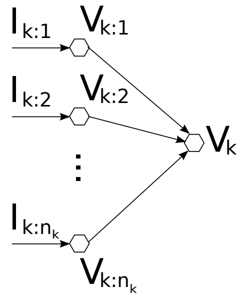

Figure 1.5.1: A number of reference nodes to be merged into one.

Note that the port current, $I_{p,k}$, is related to the nodal currents

$$\mathbf{I}_P = \mathbf{I} = -\mathbf{I}_R$$

we thus have

$$[\mathbf{Z}_P][\mathbf{I}] = [\mathbf{Z}_P][\mathbf{I}_P] = \begin{bmatrix} \mathbf{U} & -\mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{V}_P \\ \mathbf{V}_R \end{bmatrix}$$

and

$$[\mathbf{Z}_P][\mathbf{I_R}] = -[\mathbf{Z}_P][\mathbf{I}_P] = \begin{bmatrix} -\mathbf{U} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{V}_P \\ \mathbf{V}_R \end{bmatrix}$$

therefore

$$\begin{bmatrix} \mathbf{Z}_P & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_P \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \mathbf{U} & -\mathbf{U} \\ -\mathbf{U} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{V}_P \\ \mathbf{V}_R \end{bmatrix}$$

or

$$\begin{bmatrix} \mathbf{I} \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_P & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_P \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{U} & -\mathbf{U} \\ -\mathbf{U} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{V}_P \\ \mathbf{V}_R \end{bmatrix}$$

The new extended admittance matrix is now

$$\mathbf{Y}_E = \begin{bmatrix} \mathbf{Y}_P & 0 \\ 0 & \mathbf{Y}_P \end{bmatrix} \begin{bmatrix} \mathbf{U} & -\mathbf{U} \\ -\mathbf{U} & \mathbf{U} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_P & -\mathbf{Y}_P \\ -\mathbf{Y}_P & \mathbf{Y}_P \end{bmatrix}.$$

Now, convert back to the scattering matrix

$$\mathbf{S}_E = (\mathbf{U} - Z_0\mathbf{Y}_E)(\mathbf{U} + Z_0\mathbf{Y}_E)^{-1} = \begin{bmatrix} \mathbf{U} - Z_0\mathbf{Y}_P & Z_0\mathbf{Y}_P \\ Z_0\mathbf{Y}_P & \mathbf{U} - Z_0\mathbf{Y}_P \end{bmatrix} \begin{bmatrix} \mathbf{U} + Z_0\mathbf{Y}_P & -Z_0\mathbf{Y}_P \\ -Z_0\mathbf{Y}_p & \mathbf{U} + Z_0\mathbf{Y}_P \end{bmatrix}^{-1}$$

where

$$Z_0\mathbf{Y}_P = (\mathbf{U} + \mathbf{S})^{-1}(\mathbf{U} - \mathbf{S}).$$

1.5 Combining Shared Reference Nodes

The above derivation assumes that each original port has its own local reference node (LRN), resulting in the extended network to have the matrix order doubling the original representation. In reality, multiple ports may share one reference node. The derivation below shows how to merge multiple LRN's into one. A treatment and the LRN concept can be found in [**?**]. Application of the LRN concep is discussed in [4]. Due to the importance and difficulty of the subject, an independent derivation is made below.

Assuming certain (say n_k in number) nodes are merged into one, with the same potential (or voltage) V_k. For clarity we re-index the nodes such that the n_k reference nodes (to be nerged) are bundled at the end. The corresponding admittance equation can be written in the partitioned form as

$$\begin{bmatrix} \mathbf{Y}_{aa} & \mathbf{Y}_{ak} \\ \mathbf{Y}_{ka} & \mathbf{Y}_{kk} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_k \end{bmatrix} = \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_k \end{bmatrix} \tag{1.5.1}$$

where

$$\mathbf{I}_k = \begin{bmatrix} I_{k:1} & I_{k:2} & \ldots & I_{k:n_k} \end{bmatrix}^T$$

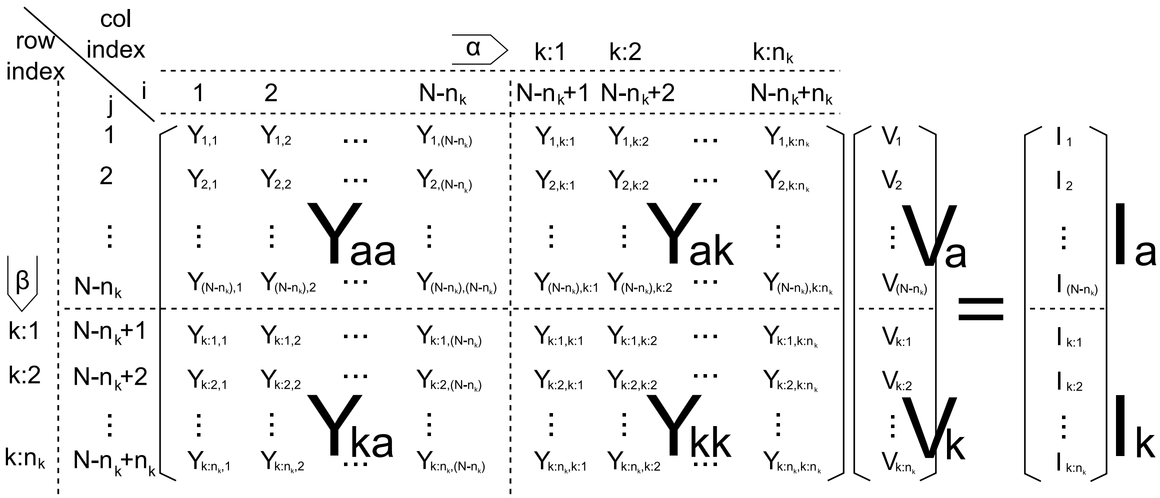

Figure 1.5.2: Indexing scheme for partitioning the nodal matrix.

and

$$\mathbf{V}_k = [\ V_{k:1} \quad V_{k:2} \quad ... \quad V_{k:n_k}\ ]^T.$$

The nodal equations in block form are

$$\begin{aligned} \mathbf{Y}_{aa}\mathbf{V}_a + \mathbf{Y}_{ak}\mathbf{V}_k &= \mathbf{I}_a \\ \mathbf{Y}_{ka}\mathbf{V}_a + \mathbf{Y}_{kk}\mathbf{V}_k &= \mathbf{I}_k \end{aligned} \qquad (1.5.2)$$

The first block equation can be written as

$$\sum_{m=1}^{N-n_k} Y_{i,m}V_m + \sum_{\beta=1}^{n_k} Y_{i,k:\beta}V_{k:\beta} = I_i, \qquad i = 1, 2, ..., (N - n_k)$$

Knowing

$$V_{k:\beta} = V_k, \qquad \beta = 1, 2, ..., n_k$$

we now have

$$\sum_{m=1}^{N-n_k} Y_{i,m}V_m + \left(\sum_{\beta=1}^{n_k} Y_{i,k:\beta}\right) V_k = I_i, \qquad i = 1, 2, ..., (N - n_k). \qquad (1.5.3)$$

On the other hand, the second equation is equivalent to

$$\sum_{i=1}^{N-n_k} Y_{k:j,i}V_i + \sum_{\beta=1}^{n_k} Y_{k:j,k:\beta}V_{k:\beta} = I_{k:j}, \qquad j = 1, 2, ..., n_k$$

by summing over all $k : j$ yields

$$\sum_{j=1}^{n_k}\sum_{i=1}^{N-n_k} Y_{k:j,i}V_i + \sum_{j=1}^{n_k}\sum_{\beta=1}^{n_k} Y_{k:j,k:\beta}V_{k:\beta} = \sum_{j=1}^{n_k} I_{k:j}, \qquad j = 1, 2, ..., n_k$$

If we denote

$$I_{\bar{k}} \equiv \sum_{j=1}^{n_k} I_{k:j}$$

noticing that$V_{k:\beta} = V_k$

$$\sum_{i=1}^{N-n_k}\left(\sum_{j=1}^{n_k} Y_{k:j,i}\right) V_i + \left(\sum_{j=1}^{n_k}\sum_{\beta=1}^{n_k} Y_{k:j,k:\beta}\right) V_k = \sum_{j=1}^{n_k} I_{k:j} = I_{\bar{k}}, \qquad j = 1, 2, ..., n_k \qquad (1.5.4)$$

Denote a single-column vector

$$\mathbf{Y}_{ak}^{r} = \begin{bmatrix} Y_{c:1}^{r} \equiv \sum_{\beta=1}^{n_k} Y_{1,k:\beta} \\ Y_{c:2}^{r} \equiv \sum_{\beta=1}^{n_k} Y_{2,k:\beta} \\ \vdots \\ Y_{c:(N-n_k)}^{r} \equiv \sum_{\beta=1}^{n_k} Y_{N-n_k,k:\beta} \end{bmatrix},$$

a single-row vector

$$\mathbf{Y}_{ka}^{r} = \begin{bmatrix} Y_{r:1}^{r} \equiv \sum_{j=1}^{n_k} Y_{k:j,1}, & Y_{r:2}^{r} \equiv \sum_{j=1}^{n_k} Y_{k:j,2}, & \ldots, & Y_{r:(N-n_k)}^{r} \equiv \sum_{j=1}^{n_k} Y_{k:j,N-n_k} \end{bmatrix}$$

and a scalar

$$Y_{kk}^{r} \equiv \sum_{\alpha=1}^{n_k} \sum_{\beta=1}^{n_k} Y_{k:\alpha,k:\beta}$$

Then the initial block form 1.5.2 is reduced to

$$\begin{aligned} \mathbf{Y}_{aa}\mathbf{V}_a + \mathbf{Y}_{ak}^{r} V_k &= \mathbf{I}_a \\ \mathbf{Y}_{ka}^{r}\mathbf{V}_a + \mathbf{Y}_{kk}^{r} V_k &= I_k \end{aligned} \tag{1.5.5}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{Y}_{aa} & \mathbf{Y}_{ak}^{r} \\ \mathbf{Y}_{ka}^{r} & Y_{kk}^{r} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ V_k \end{bmatrix} = \begin{bmatrix} \mathbf{I}_a \\ I_k \end{bmatrix}. \tag{1.5.6}$$

The recipe provided here can be applied repeatedly for more groups of shared reference nodes

Chapter 2

S-Parameter Generation from Transmission-line Models

2.1 Basic Transmission-line Theory

Real world aplications, for transmiting electrical and electronic signals, often involve the use of (N+1) conducting wires with N carrying signals and one being the reference line. A formal derivation of the governing equations for such problem can be found in [8] by solving the Maxwell's equation under certain assumptions and/or approximations. The cannonical problem assumes (N+1) perfect lossless conducting rods in parallel, of infinite length and constant cross sections. This is illustrated by Figure 2.1.1. To set up for further analysis, a Cartesian coordinate system is installed such that the z-direction is aligned with all the conductors.

Most of the results can be explined by looking only at one cross section, as shown in Figure 2.1.2. The assumption is: the electromagnetic field variables, $\vec{E}$ the electric field and $\vec{H}$ the magnetic field intensity, have ZERO vector components along the longitudinal or the z-direction. As time varies, the electric and magnetic field variables have only the transverse components: they are perpendicular to each other and both are perpendicular to the z-direction at all time. This is referred to as the transverse electromagnetic (TEM) field. Below are the properties (or consequencies) of the TEM field and other related varibles associated with the (N+1) conductors:

- First, determine the solutions for the in-plane normalized potential functions $\{\phi_i(x, y);\, i = 1, 2, ..., N\}$, one for each of the conductors labeled 1 through N:

  $$\nabla^2 \phi_i(x, y) = 0,\, i = 1, 2, ..., N.$$

  The 2D coordinate vector can also be written as

  $$\vec{\rho} \equiv x\hat{x} + y\hat{y}$$

  where $\hat{x}$ and $\hat{y}$ are the unit vector along the x and y direction, respectively. Thus, it is equivalent to use $\phi_i(\vec{\rho})$ to mean $\phi_i(x, y)$ and *vice versa*.

- Each of the $\{\phi_i(x, y)\}$ function assumes constant values on all conductor surfaces.

- Denote $\phi_i(j)$ to be the constant value of $\phi_i(x, y)$ when (x, y) takes on values associated with the surface of Conductor-j, then

  $$\phi_i(j) = \delta_{ij} + \phi_i(0),\, i = 1, 2, ..., N$$

  where $\delta_{ij} = 1$ only when $i = j$ and 0 otherwise.

- The per-unit-length (p.u.l.) capacitance constants are determined by

  $$C_{ij} = -\epsilon \oint_{l_i} \hat{n}_i \cdot \nabla \phi_j(x, y) dl_i$$

  where the integration path is around the countour representing Conductor-i, as shown in Figure 2.1.2

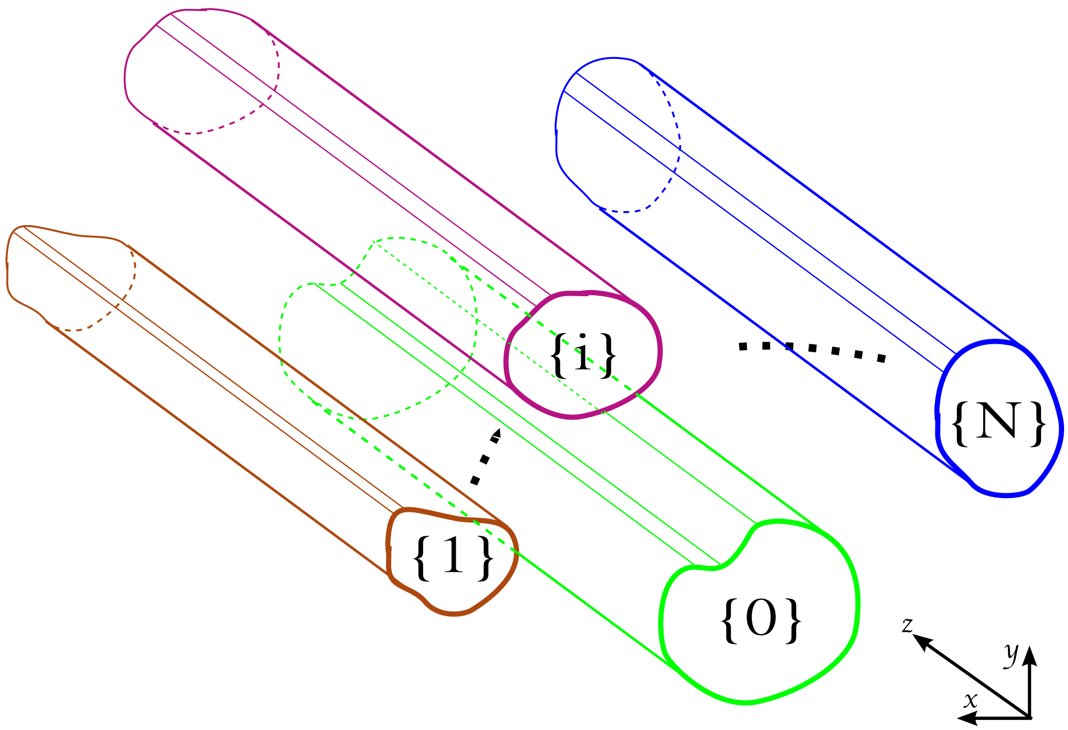

Figure 2.1.1: (N+1) parallel conducting rods of infinite length and constant cross-sections.

- The p.u.l inductance constants, L_{jk}, can be determined by

$$\sum_{j=1}^{N} C_{ij} L_{jk} = \epsilon\mu\delta_{ik},\; i = 1, 2, ..., N;\; k = 1, 2, ..., N$$

- Each of the normalized potential function satisfies

$$\lim_{|\vec{\rho}|\to\infty} [\vec{\rho} \cdot \nabla\phi_i(\vec{\rho})] = 0$$

- Definition of voltages

$$V_i(z, t) = -\int_0^j \vec{E} \cdot d\vec{l}$$

where the integration path starts on the surface of Conductor-0, and ends on the surface of Conductor-i. Due to the conservative nature of the field, the resultant integral yields the same value regardless how the path is chosen.

- Defininition of currents

$$I_i(z, t) = \oint_i \vec{H} \cdot \hat{t}_i dl_i$$

- The electric field and magnetic field variables are associate with the voltages and currents by

$$\begin{aligned} \vec{E} &= -\sum_{i=1}^{N} V_i(z, t)\nabla\phi_i(\vec{\rho}) \\ \vec{H} &= -\sum_{i=1}^{N} \left[\frac{1}{\mu} \sum_{j=1}^{N} L_{ij} I_j(z, t) \right] \hat{k} \times \nabla\phi_i(\vec{\rho}) \end{aligned}$$

- The p.u.l capacitance and inductance constants satisfy the Telegrapher's equation in lossless medium

$$\begin{cases} -\frac{\partial V_i}{\partial z} = \sum_{j=1}^{N} L_{ij} \frac{\partial I_j}{\partial t}, & i = 1, 2, ..., N \\ -\frac{\partial I_i}{\partial z} = \sum_{j=1}^{N} C_{ij} \frac{\partial V_j}{\partial t} & j = 1, 2, ..., N \end{cases} \tag{2.1.1}$$

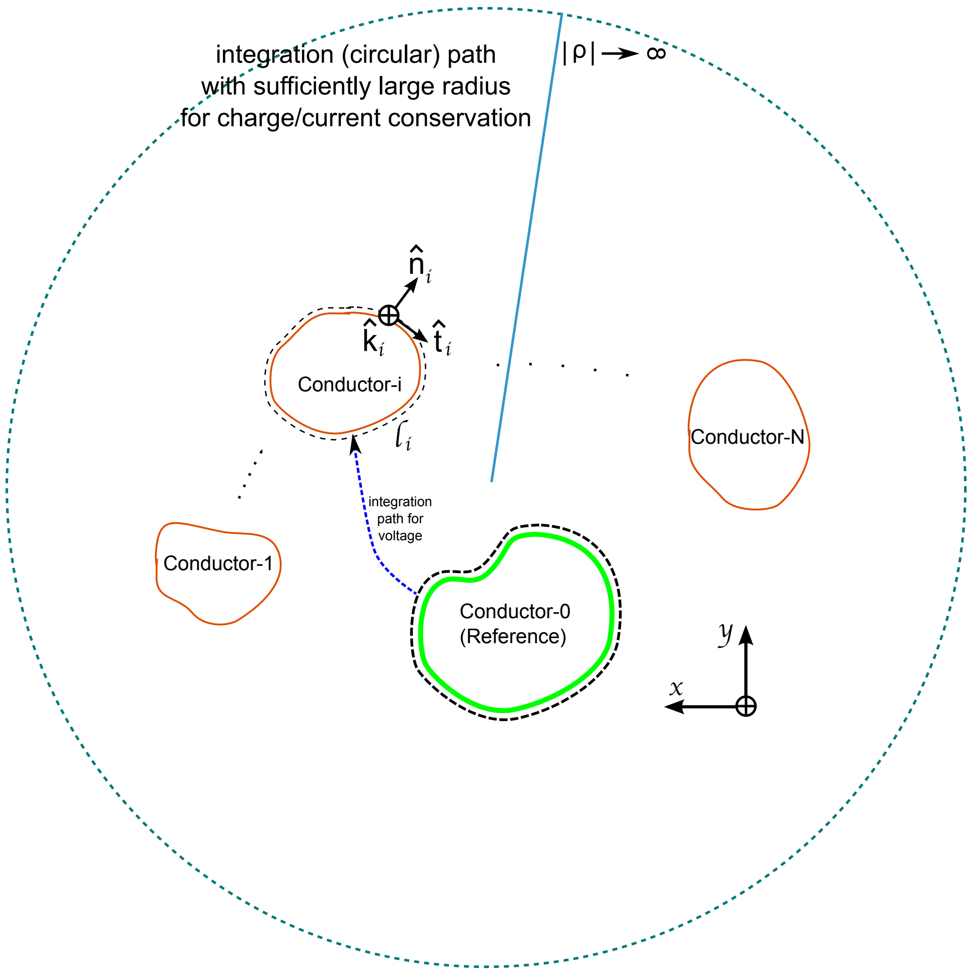

Figure 2.1.2: Cross sectional view of a transmission-line system of (N+1) conductors.

- The p.u.l charge variables are given by

$$q_i(z,t) = \frac{\epsilon}{\Delta z}\int_z^{z+\Delta z} dz \oint_i \vec{E}\cdot\hat{n}_i dl_i = \sum_{j=1}^{N} C_{ij}V_j(z,t)$$

- The p.u.l magnetic flux

$$\Phi_i(z,t) = \frac{\mu}{\Delta z}\int_z^{z+\Delta z} dz \int_i^0 \vec{H}\cdot\hat{k}\times\vec{dl} = \sum_{j=1}^{N} L_{ij}I_j(z,t)$$

- The p.u.l energy

$$W(z,t) = \frac{1}{2}(\epsilon\vec{E}\cdot\vec{E} + \mu\vec{H}\cdot\vec{H}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(L_{ij}I_iI_j + C_{ij}V_iV_j)$$

- Conservation (continuity) of charge and current in a source-free environment

$$0 = \oint \vec{H}\cdot\hat{t}dl = I_0(z,t) + \sum_{j=1}^{N} I_j(z,t)$$

where the integration contour is shown as the large circle enclosing all (N+1) conductors. This shows that the current flows on the reference conductor is

$$I_0(z,t) = -\sum_{j=1}^{N} I_j(z,t)$$

The time-domain telegrapher's equation of the lossless medium can be transformed into the frequency-domain version as

$$\begin{aligned}\frac{d}{dz}\mathbf{V}(z) &= -\mathbf{Z}_u\mathbf{I}(z),\\ \frac{d}{dz}\mathbf{I}(x) &= -\mathbf{Y}_u\mathbf{V}(z).\end{aligned} \tag{2.1.2}$$

with

$$\begin{aligned}\mathbf{V}(z) &\equiv [\ V_1,\ \ V_2,\ \ \ldots\ \ V_N\ ]^T\\ \mathbf{I}(z) &\equiv [\ I_1,\ \ I_2,\ \ \ldots\ \ I_N\ ]^T\end{aligned}$$

and the $\mathbf{Y_u}$, $\mathbf{Z_u}$ matrices are defined by

$$\begin{aligned}\mathbf{Z}_u &= \mathbf{R} + j\omega\mathbf{L}\\ \mathbf{Y}_u &= \mathbf{G} + j\omega\mathbf{C}\end{aligned} \tag{2.1.3}$$

where **L** and **C** are the matrix form of the inductance and capacitance constant discussed above. The **R** and **G** matrices represent the resistance and conductance factor in lossy media.

Remark. It should be noted that an (N+1) parallel conductor system is often called an N-conductor coupled transmission-line, with one count implicitly reserved for the reference conductor. Therefore, a "single transmission-line" is physically realized with 2 conductors, a "coupled transmission-lines" structure actually involves 3 conductors, and so on.

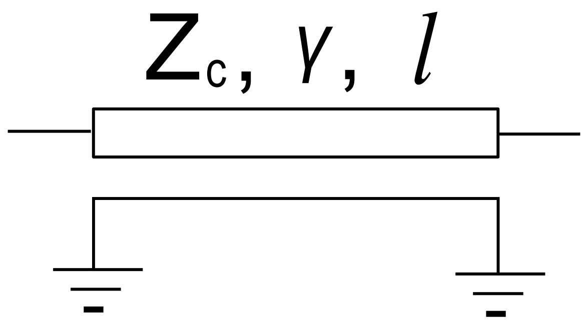

Figure 2.2.1: Generic transmission line model.

2.2 Single Transmission-line Models

2.2.1 S-Parameters Calculating of a General Lossy Transmsiion-line Segment

The ABCD matrix of a lossy transmission-line with characteristic impedance Z_c segment is [2]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \tag{2.2.1}$$

Using the well-known conversion formula

$$\begin{aligned} S_{11} &= \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \\ S_{12} &= \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \\ S_{21} &= \frac{2}{A + B/Z_0 + CZ_0 + D} \\ S_{22} &= \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \end{aligned}$$

where Z_0 is the reference impedance, which may not be the same as Z_c. Denote

$$\xi = \frac{Z_c}{Z_0} \tag{2.2.2}$$

then,

$$\begin{aligned} & A + B/Z_0 + CZ_0 + D \\ & = \cosh(\gamma l) + \frac{Z_c}{Z_0}\sinh(\gamma l) + \frac{Z_0}{Z_c}\sinh(\gamma l) + \cosh(\gamma l) \\ & = 2\cosh(\gamma l) + \sinh(\gamma l)(\xi + \xi^{-1}) \end{aligned}$$

and

$$\begin{aligned} S_{11} = S_{22} &= \frac{\sinh(\gamma l)(\xi - \xi^{-1})}{2\cosh(\gamma l) + \sinh(\gamma l)(\xi + \xi^{-1})} \\ S_{12} = S_{21} &= \frac{2}{2\cosh(\gamma l) + \sinh(\gamma l)(\xi + \xi^{-1})} \end{aligned}$$

Therefore, for given $\{Z_c, \gamma l, Z_0\}$, the S-parameters of the two-port network associated with the transmission-line segment is determined by the above equations. In a special case of $Z_0 = Z_C$,

$$\begin{aligned} S_{11} &= S_{22} = 0 \\ S_{12} &= S_{21} = \frac{1}{\cosh(\gamma l) + \sinh(\gamma l)} = \exp(-\gamma l). \end{aligned}$$

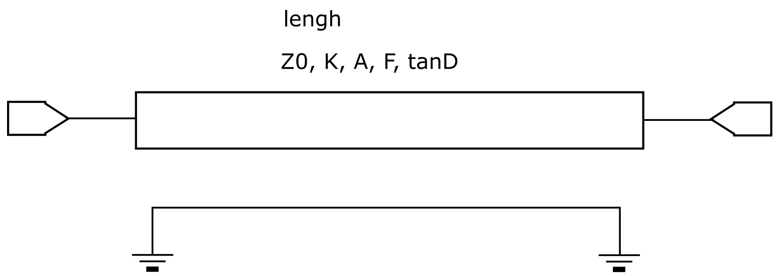

Figure 2.2.2: ADS Transmission line model parameters.

2.2.2 H-SPICE W-Element Model

H-SPICE [7] provides W-element model which describes the frequency dependent per-unit-length R/G parameters by

$$R(f) = R_0 + \sqrt{f}(1+j)R_s$$
$$G(f) = G_0 + \frac{f}{\sqrt{1+(f/f_{gd})^2}}G_d$$

where the default value for f_{gd} is zero.

The assumed relations are

$$R + j\omega L = R_0 + \sqrt{f}(1+j)R_s + j\omega L_0$$
$$G + j\omega C = G_0 + \frac{f}{\sqrt{1+(f/f_{gd})^2}}G_d + j\omega C_0$$

In other words

$$R = R_0 + \sqrt{f}R_s$$
$$L = \frac{R_s}{2\pi\sqrt{f}} + L_0$$
$$G = G_0 + \frac{f}{\sqrt{1+(f/f_{gd})^2}}G_d$$
$$C = C_0$$

Either $G_d = 0$ or $f_{gd} = 0$ will make $G = G_0$, when $f_{gd} = \infty$, $G = G_0 + fG_d$.

Under the assumptions

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$= \sqrt{\frac{R_0 + \sqrt{f}(1+j)R_s + j\omega L_0}{G_0 + \frac{f}{\sqrt{1+(f/f_{gd})^2}}G_d + j\omega C_0}}$$

There is a confusing catch. HSPICE takes "$f_{gd} = 0$" as a software-flag for ignoring the $\frac{1}{\sqrt{1+(f/f_{gd})^2}}$ term (treating it as unity) which in fact is equivalent to saying "$f_{gd} = \infty$".

2.2.3 ADS Transmission-line Model TLINP

The Agilent ADS [6] simulation environment provides several transmission-line models for 2-port network. Among those transmission-line elements, the most comprehensive one is the so-called physical transmission-line or TLINP.

where

- Z_0^∞ is the characteristic impedance ignoring loss

- *length* is the physical length
- K is the effective dielectric constant
- A is the conductor loss factor, in dB/m
- F is the reference frequency used in calculating the frequency depedent loss
- $\tan\delta$ is loss tangent

$$\alpha_c \times (20\log_{10} e) = \begin{cases} A & \text{if } F = 0 \\ A\sqrt{\frac{f}{F}} & \text{if } f > 0 \end{cases}$$

and the shunt admittance is

$$G = \omega C \tan\delta = 2\pi f C \tan\delta = f G_d$$

which implies

$$G_d = 2\pi C \tan\delta$$

On the other hand,

$$\begin{aligned} \alpha + j\beta = \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= j\omega\sqrt{LC}\sqrt{(\frac{R}{j\omega L} + 1)(\frac{G}{j\omega C} + 1)} \\ &\approx j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right] \\ &= \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + j\omega\sqrt{LC} \end{aligned}$$

Thus,

$$\begin{aligned} \beta &= \omega\sqrt{LC} \\ \alpha &= \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \\ &= \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + 2\pi f \tan\delta\right) \\ &= \frac{R}{2Z_0^\infty} + \pi f\sqrt{LC}\tan\delta \\ &= \alpha_c + \alpha_d \end{aligned}$$

where

$$Z_0^\infty = \sqrt{\frac{L}{C}}$$

The total loss contains both conductor loss α_c and the dielectric loss α_d. It is obvious that

$$\alpha_d = \pi f\sqrt{LC}\tan\delta$$

and

$$\alpha_c = \frac{R}{2Z_0^\infty}$$

When including the skin effect

$$R = R_{dc} + R_s\sqrt{f}$$

then

$$\begin{aligned} \alpha_c &= \frac{R_{dc} + R_s\sqrt{f}}{2Z_0} \\ &= \frac{R_{dc}}{2Z_0} + \frac{R_s}{2Z_0}\sqrt{f} \end{aligned}$$

If the loss function A is set to

$$A = A(f) = \frac{A_0}{\sqrt{f}} + A_1 \tag{2.2.3}$$

then

$$\begin{aligned}\alpha_c(20\log_{10} e) &= (\frac{A_0}{\sqrt{f}} + A_1)\sqrt{\frac{f}{F}} \\ &= \frac{A_0}{\sqrt{F}} + \frac{A_1}{\sqrt{F}}\sqrt{f}\end{aligned}$$

which can be used to determine $\{R_{dc}, R_s\}$ by

$$\begin{aligned}\frac{R_{dc}}{2Z_0^\infty} &= \frac{A_0}{(20\log_{10} e)\sqrt{F}} \\ \frac{R_s}{2Z_0^\infty} &= \frac{A_1}{(20\log_{10} e)\sqrt{F}}\end{aligned}$$

Or

$$\begin{aligned}R_{dc} &= 2Z_0^\infty \frac{A_0}{(20\log_{10} e)\sqrt{F}} \\ R_s &= 2Z_0^\infty \frac{A_1}{(20\log_{10} e)\sqrt{F}}\end{aligned}$$

Approximation of impedance

$$\begin{aligned}Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= \sqrt{\frac{L}{C}}\sqrt{\frac{1 + \frac{R}{j\omega L}}{1 + \frac{G}{j\omega C}}} \\ &\approx \sqrt{\frac{L}{C}}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 - \frac{G}{j\omega C}\right)} \\ &\approx \sqrt{\frac{L}{C}}\sqrt{1 + \frac{1}{j\omega}\left(\frac{R}{L} - \frac{G}{C}\right)} \\ &\approx \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2}\frac{1}{j\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\right]\end{aligned}$$

2.2.4 Conversion from ADS TLINP to HSPICE W-Element Model

For a given set of TLINP model parameters

- Z_0^∞ is the characteristic impedance ignoring loss
- *length* is the physical length
- K is the effective dielectric constant
- A is the loss function, in particular

  $$A = A(f) = \frac{A_0}{\sqrt{f}} + A_1$$

  where A_0 is the conductor loss factor due to resistivity, in dB/m, and A_s is the conductor loss factor due to skin effect, in dB/m
- F is the reference frequency used in calculating the frequency depedent loss

- $\tan\delta$ is loss tangent

the corresponding W-element parameters can be calculated by

$$\begin{aligned}
&R_0 = R_{dc} = 2Z_0^{\infty}\frac{A_0}{(20\log_{10}e)\sqrt{F}}\\
&R_s = 2Z_0^{\infty}\frac{A_1}{(20\log_{10}e)\sqrt{F}}\\
&L = \frac{Z_0^{\infty}\sqrt{K}}{c_0}\\
&G_0 = 0\\
&G_d = 2\pi C\tan\delta\\
&C = \frac{\sqrt{K}}{c_0 Z_0^{\infty}}
\end{aligned}$$

Conversely, knowing $\{R_0, R_s, L, G_0, G_d, C\}$, the TLINP parameters are obtained by fixing F to be a constant, say 1 GHz

$$\begin{aligned}
&Z_0^{\infty} = \sqrt{\frac{L}{C}}\\
&K = c_0^2 LC\\
&A_0 = \frac{R_0(20\log_{10}e)}{2Z_0^{\infty}}\sqrt{F}\\
&A_1 = \frac{R_s(20\log_{10}e)}{2Z_0^{\infty}}\sqrt{F}\\
&\tan\delta = \frac{G_d}{2\pi C}
\end{aligned}$$

assuming the coductor loss is defined by

$$(20\log_{10}e)\alpha_c = (\frac{A_0}{\sqrt{f}} + A_1)\sqrt{\frac{f}{F}},$$

and the constant

$$(20\log_{10}e) = 8.6859.$$

2.2.5 Lossless Single-line Model

when

$$R = 0,\ G = 0,$$

$$\gamma = j2\pi f\sqrt{L_0C_0},\ \ Z_c = \sqrt{\frac{L_0}{C_0}}$$

$$\begin{aligned}
\cosh(\gamma l) &= \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{e^{j\beta l} + e^{-j\beta l}}{2} = \cos(\beta l),\\
\cosh(\gamma l) &= \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{e^{j\beta l} - e^{-j\beta l}}{2} = j\sin(\beta l),
\end{aligned}$$

than

$$\mathbf{S} = \frac{1}{2\cos(\beta l) + j\sin(\beta l)(\xi + \xi^{-1})}\begin{bmatrix} j\sin(\beta l)(\xi - \xi^{-1}) & 2\\ 2 & j\sin(\beta l)(\xi - \xi^{-1})\end{bmatrix}$$

where

$$\beta = 2\pi f\sqrt{L_0C_0}.$$

2.3 Multiple Coupled Transmission-line Model

2.3.1 Fundamental equation

Methods for deriving S-parameters from multi-conductor transmission-line parameters can be found in [10]. The telegrapher's equations governing for the multiconductor transmission lines are

$$\begin{aligned}
\frac{d}{dx}\mathbf{V}(x) &= -\mathbf{Z}_u\mathbf{I}(x),\\
\frac{d}{dx}\mathbf{I}(x) &= -\mathbf{Y}_u\mathbf{V}(x).
\end{aligned}$$

The $\mathbf{Y}$, $\mathbf{Z}$ matrices are related to the RLGC matrices by

$$\begin{aligned}\mathbf{Z}_u &= \mathbf{R} + j\omega\mathbf{L},\\ \mathbf{Y}_u &= \mathbf{G} + j\omega\mathbf{C}.\end{aligned}$$

where the longitudinal coordinate variable z in Eqn.(2.1.2) is replaced by x.

2.3.2 Method 1

According to [9], the solutions are

$$\begin{aligned}\mathbf{V}(x) &= \exp(-\mathbf{\Gamma}x)\mathbf{A} + \exp(\mathbf{\Gamma}x)\mathbf{B},\\ \mathbf{Y}_0^{-1}\mathbf{I}(x) &= \exp(-\mathbf{\Gamma}x)\mathbf{A} - \exp(\mathbf{\Gamma}x)\mathbf{B},\end{aligned}$$

where

$$\begin{aligned}\mathbf{\Gamma} &= \sqrt{\mathbf{ZY}} = \mathbf{P}\gamma\mathbf{P}^{-1},\\ \mathbf{Y}_0 &= \mathbf{Z}^{-1}\mathbf{\Gamma} = \mathbf{Y}\mathbf{\Gamma}^{-1}.\end{aligned}$$

$\mathbf{P}$ is the eigenvector of $(\mathbf{ZY})$, it is also the eigenvector of $\mathbf{\Gamma}$. $\gamma's$ are. From the properties of functions of matrices

$$\exp(-\mathbf{\Gamma}x) = \mathbf{P}\exp(-\gamma\mathbf{x})\mathbf{P}^{-1}.$$

The $(N+1)$-conductor transmission lines of length d can be treated as a $(2N)$-port network, having N ports on the sending side and N ports on the receiving side. The admittance equation can be shown as

$$\begin{bmatrix}\mathbf{I}_S\\ \mathbf{I}_R\end{bmatrix} = \begin{bmatrix}\mathbf{Y}_0\coth(\mathbf{\Gamma}d) & -\mathbf{Y}_0\text{csch}(\mathbf{\Gamma}d)\\ -\mathbf{Y}_0\text{csch}(\mathbf{\Gamma}d) & \mathbf{Y}_0\coth(\mathbf{\Gamma}d)\end{bmatrix}\begin{bmatrix}\mathbf{V}_S\\ \mathbf{V}_R\end{bmatrix}$$

where

$$\coth(\mathbf{\Gamma}d) = \mathbf{P}\coth(\gamma)\mathbf{P}^{-1}$$

, the solution to the telegrapher's equations are

$$\begin{bmatrix}\mathbf{V}(x)\\ \mathbf{I}(x)\end{bmatrix} = \exp\left\{-x\begin{bmatrix}0 & \mathbf{Z}_u\\ \mathbf{Y}_u & 0\end{bmatrix}\right\}\begin{bmatrix}\mathbf{V}(0)\\ \mathbf{I}(0)\end{bmatrix}.$$

It can be shown that

$$\begin{aligned}\exp\left\{-x\begin{bmatrix}0 & \mathbf{Z}_u\\ \mathbf{Y}_u & 0\end{bmatrix}\right\} &= \begin{bmatrix}\Omega & \alpha\\ \beta & \Omega^T\end{bmatrix}\\ &= \begin{bmatrix}\phi_1(x^2\mathbf{Z}_u\mathbf{Y}_u) & -x\mathbf{Z}_u\phi_2(x^2\mathbf{Y}_u\mathbf{Z}_u)\\ -x\mathbf{Y}_u\phi_2(x^2\mathbf{Z}_u\mathbf{Y}_u) & \phi_1(x^2\mathbf{Y}_u\mathbf{Z}_u)\end{bmatrix}\end{aligned}$$

where the ϕ_1, ϕ_2 functions are defined by

$$\begin{aligned}\phi_1(\xi) &= \sum_{k=0}^{\infty}\frac{\xi^k}{(2k)!} = \cosh(\sqrt{\xi}),\\ \phi_2(\xi) &= \sum_{k=0}^{\infty}\frac{\xi^k}{(2k+1)!} = \frac{\sinh(\sqrt{\xi})}{\sqrt{\xi}}.\end{aligned}$$

Let

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{V}(0), \mathbf{V}_2 = \mathbf{V}(x),\\ \mathbf{I}_1 &= \mathbf{I}(0), \mathbf{I}_2 = \mathbf{I}(x),\end{aligned}$$

then the impedance matrix is

$$\mathbf{Z} = \begin{bmatrix}-\beta^{-1}\Omega^T & -\beta^{-1}\\ \alpha - \Omega\beta^{-1}\Omega^T & -\Omega\beta^{-1}\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{A}\end{bmatrix}$$

which satisfies

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}.$$

Once the impedance function is obtained, the S-parameters can be derived by

$$\mathbf{S} = \mathbf{I}_{2N} - 2(\mathbf{Z}/Z_0 + \mathbf{I}_{2N})^{-1}.$$

Conversely, if the (2N)-port S-parameters are known, we like to recover the transmission-line parameters. The initial steps involve finding the $\{\mathbf{A}, \mathbf{B}\}$ matrices by

$$\beta = -\mathbf{B}^{-1},\ \Omega = \mathbf{A}\mathbf{B}^{-1},\ \alpha = \mathbf{B} - \mathbf{A}\mathbf{B}^{-1}\mathbf{A},$$

or

$$\begin{aligned} \mathbf{A}\mathbf{B}^{-1} &= \phi_1(x^2 \mathbf{Z}_u \mathbf{Y}_u), \\ \mathbf{B}^{-1} &= x\mathbf{Y}_u \phi_2(x^2 \mathbf{Z}_u \mathbf{Y}_u). \end{aligned}$$

If the scattering matrix is given as

$$\mathbf{S} = \begin{bmatrix} \mathbf{U} & \mathbf{V} \\ \mathbf{V} & \mathbf{U} \end{bmatrix}$$

where both $\{\mathbf{U}, \mathbf{V}\}$ are symmetric, that are tied to the impedance matrix with

$$\mathbf{Z} = Z_0[-\mathbf{I}_{2N} + 2(\mathbf{I}_{2N} - \mathbf{S})^{-1}]$$

implying

$$\begin{aligned} \mathbf{A} &= Z_0 \left\{ -\mathbf{I}_N + 2\left[\mathbf{I}_N - \mathbf{U} - \mathbf{V}(\mathbf{I}_N - \mathbf{U})^{-1}\mathbf{V}\right]^{-1} \right\}, \\ \mathbf{B} &= 2Z_0 \left[-\mathbf{V} + (\mathbf{I}_N - \mathbf{U})V^{-1}(\mathbf{I}_N - \mathbf{U}) \right]^{-1}. \end{aligned}$$

More details can be found in [10].

Chapter 3

S-Parameter Generation by Composition of a Sub-Circuit

S-parameters are initially developed to solve RF/microwave engineering problems, are nowadayincreasingly popular in the area of signal integrity as devices and systems are running at high-speeds that are encroaching into the traditional RF/microwave territory. More and more components, especially passive interconnects, are readily characterized by S-parameters. When the S-parameter models are connected in a real system model, it is preferrable to have a means to compute the overall combined S-parameter block directly without going through the modified nodal analysis (MNA). In fact, a formulation introduced by[12] can be applied.

3.1 The "Shared Reference Node" and Universal Reference Impedance Assumption for All S-Parameter Blocks

The original formulation of S-parameters built the formulation based on the concept of ports. Each port is composed of two terminals (or nodes).

The concept of ports has important physical meanings:

- In RF/microwave applications, the notion of voltage and current is well defined when the electromagnetic field is constrained to the TEM (or quasi-TEM) mode;
- A transmission-line structure is typically required to support TEM wave;
- For transmission-line, there is always a 'signal' conductor and its 'reference' conductor, which is natually mapped to the (+) and (-) nodes.

In general, an S-parameter block can have multiple reference nodes that are not necessarily the same. With some manipulations, it is possible to derive an S-parameter data block with a common reference node shared

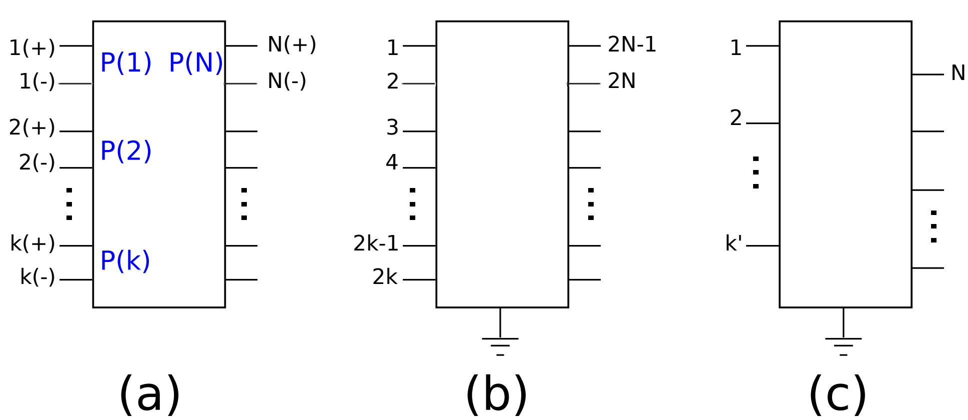

Figure 3.1.1: (a) N-port network used by the original S-parameter formulation, and (b) N-node network.

by all signal nodes. Shown in Figure 3.1.1(a), an N-port network has a total of 2N nodes, as in Figure 3.1.1(b), where the common reference node (GND) can be interpretted as the point at physical infinity. Among the 2N nodes, some of them might be sharing the common grounding (GND) structure can therefore be merged with the GND node. Thus, any N-port network can always be converted to a N'-node network where each node coincide with a port (whose reference terminal in GND) as in Figure 3.1.1(c). Obviously, $N' \leq 2N$.

In the rest of the formulation for manipulting S-parameter blocks, it is assumed that all S-blocks are readily converted to the nodal form with shared respective reference nodes per block. Furthermore, the reference nodes are the same, and labeled as GND, which has ZERO potential at all frequencies. In the strict sense, the use of infinity as GND shall always work. In practice, the notion of common GND works if all the S-blocks physically share a large, continous metal structure. In a relaxed sense, if several reference nodes are DC connected with almost no resistance, and the so-called controversial "ground-bounce" effect is not the subject of interest, they can be merged into the common GND node. Due to this assumption, the concept of port and node become equivalent, and will be used interchangeably throughout the rest of this article.

Also, for simplicity, all ports of all S-blocks are assumed to share the same port reference impedance Z_0.

3.2 Wave (Scattering Matrix) based Simulation Algorithm

3.2.1 Wave Parameters related to Voltage and Current Variables

Assuming all ports have the same real-valued characteristic impedance $Z_{0i} = Z_0$ then the wave variables are defined as

$$a_i = \frac{V_i + Z_0 I_i}{2\sqrt{Z_0}},\ b_i = \frac{V_i - Z_0 I_i}{2\sqrt{Z_0}}.$$

Extreme care is required when port impedances are allowed to be complex-valued, as discussed in Subsections 1.3.1.

3.2.2 Scattering Matrix under Renormalization of Port Impedances

The initial port impedances $\{Z_{01}, Z_{02}, ..., Z_{0n}\}$ can be renormalized to $\{Z'_{01}, Z'_{02}, ..., Z'_{0n}\}$. As shown in Sebsection 1.3.1, if $Z_{01} = Z_{02} =, ..., = Z_{0n} \equiv Z_0$, and $Z'_{01} = Z'_{02} = ..., Z'_{0n} \equiv Z'_0$,denoting

$$\gamma = \frac{Z'_0 - Z_0}{Z'_0 + Z^*_0}$$

and

$$\Gamma = \gamma \mathbf{I},$$

then the new, renormalized scattering matrix is

$$\mathbf{S}' = (\mathbf{S} - \mathbf{\Gamma})(\mathbf{I} - \mathbf{\Gamma}\mathbf{S})^{-1}.$$

It is advantageous to ensure that all networks are normalized to the same impedance, $Z_0 = 50$ unless specified otherwise.

3.3 Method of Merging Scattering Parameter Blocks

The results in this section is largely from (or inspirely by) [**?**]. Assuming all ports share a global GND node and identical impedance (Z_0). Each node thus uniquely maps to a port. Among the total of $N = N_1 + N_2$ ports, N_1 are exposed, i.e., identified by user to be shown in the final results, and N_2 are internal ports. The overall network contains M sub-networks, their respective S-parameters are known. The global wave equation can be obtained as by proper indexing

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{b}' \end{bmatrix} = \begin{bmatrix} \mathbf{S}^A & \mathbf{S}^B \\ \mathbf{S}^C & \mathbf{S}^D \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{a}' \end{bmatrix} \tag{3.3.1}$$

where

$$\mathbf{b} = \begin{bmatrix} b_1 & b_2 & \ldots & b_{N_1} \end{bmatrix}^T, \ \mathbf{a} = \begin{bmatrix} a_1 & a_2 & \ldots & a_{N_1} \end{bmatrix}^T,$$
$$\mathbf{b}' = \begin{bmatrix} b_{N_1+1} & b_{N_1+2} & \ldots & b_N \end{bmatrix}^T, \ \mathbf{a}' = \begin{bmatrix} a_{N_1+1} & a_{N_1+2} & \ldots & a_N \end{bmatrix}^T.$$

The connectivity at the internal nodes (ports) can be expressed as

$$\mathbf{b}' = \mathbf{\Lambda}\mathbf{a}' \tag{3.3.2}$$

Equations (3.3.2) and (3.3.1) and be simplified as

$$\mathbf{b} = [\mathbf{S}^A + \mathbf{S}^B(\mathbf{\Lambda} - \mathbf{S}^D)^{-1}\mathbf{S}^C]\mathbf{a}$$

Thus, the effective scattering matrix is

$$\mathbf{S}^{eff} = \mathbf{S}^A + \mathbf{S}^B(\mathbf{\Lambda} - \mathbf{S}^D)^{-1}\mathbf{S}^C. \tag{3.3.3}$$

The elements of the connectivity matrix can be determined by

$$\begin{cases} a_i^{(m:k)} + b_i^{(m:k)} \equiv \lambda_i \text{ for } P_i \text{ instances of Port } k \text{ inside sub-network } m \text{ that converge at global port } i \\ \sum_{m:k}^{P_i} a_i^{(m:k)} = \sum_{m:k}^{P_i} b_i^{(m:k)} \end{cases}$$

which can be solved by

$$\sum_{m:k}^{P_i} b_i^{(m:k)} = \sum_{m:k}^{P_i} \left[\lambda_i - b_i^{(m:k)}\right] = P_i\lambda_i - \sum_{m:k}^{P_i} b_i^{(m:k)}$$

then

$$a_i^{(m:k)} + b_i^{(m:k)} \equiv \lambda_i = \frac{2}{P_i}\sum_{m:k}^{P_i} b_i^{(m:k)} = \frac{2}{P_i}\sum_{m:k}^{P_i} a_i^{(m:k)}$$

or

$$a_i^{(m:k)} = \frac{2}{P_i}\sum_{m':k'}^{P_i} b_i^{(m':k')} - b_i^{(m:k)} = \left(\frac{2}{P_i} - 1\right) b_i^{(m:k)} + \frac{2}{P_i} \sum_{\substack{m':k' \\ m':k' \neq m:k}}^{P_i} b_i^{(m':k')}$$

Thus, all elements of the connectivity matrix can be evaluated by

$$\Lambda_{ij} = \begin{cases} \frac{1}{P_i}(2 - P_i) & i = j \\ \frac{2}{P_i} & i \neq j \end{cases}.$$

It is crucial to note that the partitions of the $\mathbf{a}|\mathbf{a}'$ *and* $\mathbf{b}|\mathbf{b}'$ *wave vectors are valid only if none nodes corresponding to (*$\mathbf{a}, \mathbf{b}$*) is a junction shared with other nodes.*

3.3.1 Illustration of the Method

As an example, a collection of S-blocks need to be combined into a new network, are shown in Figure 3.3.1. In order to apply the method defined by Equation (3.3.3), it is important that each exposed node (as one node in one of the constituent S-blocks) needs to stand alone - without connections with any other nodes.

Since there are two exposed nodes are shared by multiple nodes in the schematic of Figure 3.3.1, the merging algorithm can not be directly applied. But, the fix is simple - by using a THRU network at each of the problematic exposed nodes:

- Rename (reindex) the node. for the example in Figure 3.3.1, morphing Node-2 to Node-10, and Node-4 to Node-11
- The index morphing should apply to all original nodes involved
- Assign the initial node index to a new exposed node, and insert a THRU between this node and its morphed node

The modified network is now in the canonical form, as shown in Figure 3.3.2, to to be ready for the merging algorithm.

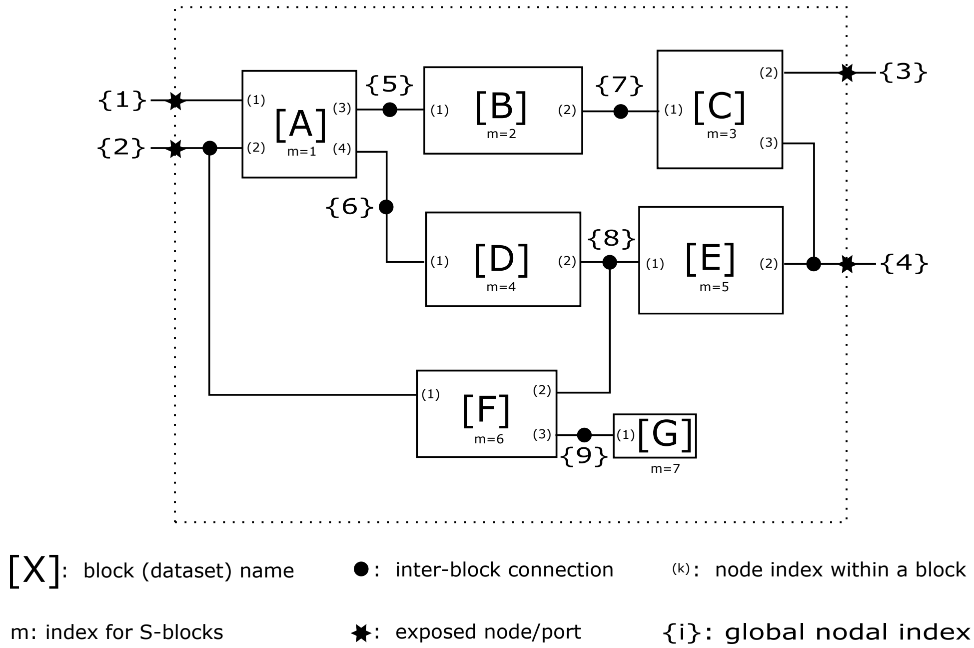

Figure 3.3.1: Multiple S-blocks to be combined into a new network.

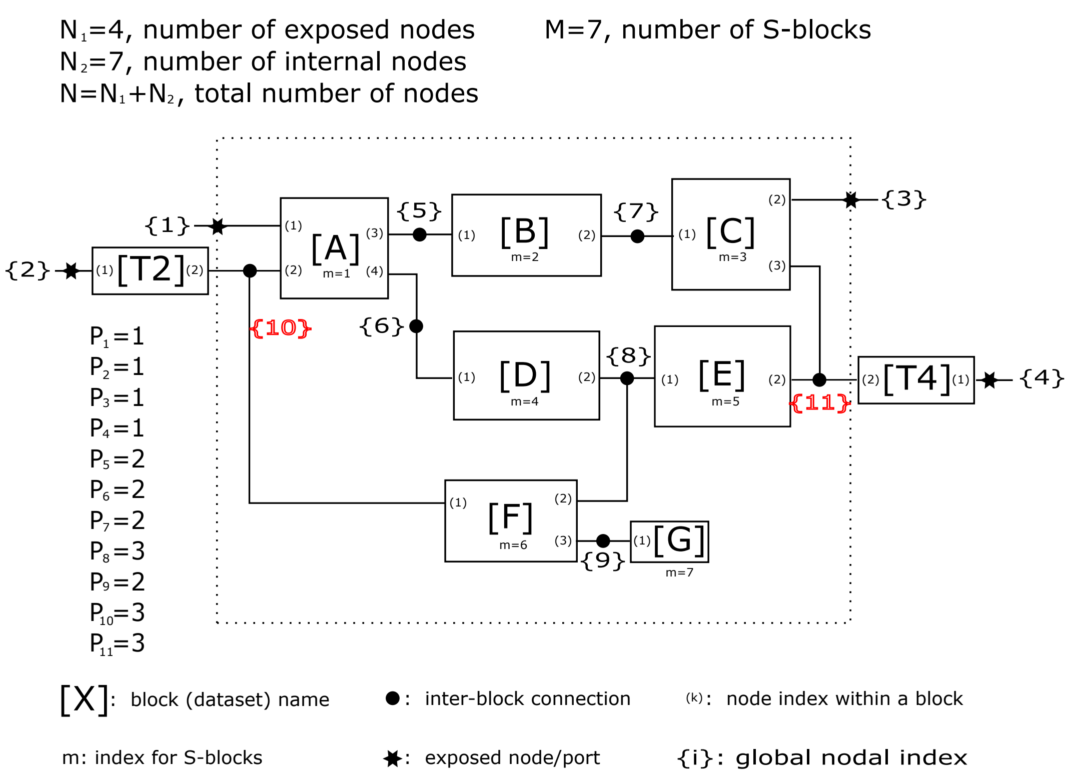

Figure 3.3.2: Canonical form of the new network definition schematic.

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